Susceptible individuals S
Infected individuals I
Recovered individuals R

$S \to I$: become infected via contact w/ infected individuals

$I \to R$: Recover at a ~~[illegible]~~ fixed rate γ

So: $\frac{1}{\gamma}$ is the average amount of time an individual is infected

Total number of individuals does not change over time $\underbrace{S+I+R=\underline{\underline{N}}}$

$$\frac{dS}{dt}, \frac{dI}{dt}, \frac{dR}{dt}$$

$$\frac{dN}{dt} = \emptyset \Rightarrow \frac{d(S+I+R)}{dt} = \emptyset$$

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = \emptyset$$

λ = the force of infection ~ the per-capita rate at which susceptible individuals acquire infection.

(*) (*) λ is not constant
λ changes with I

$$\lambda(I) = \beta \underbrace{\frac{I}{N}}$$

$\frac{I}{N}$: Interaction term

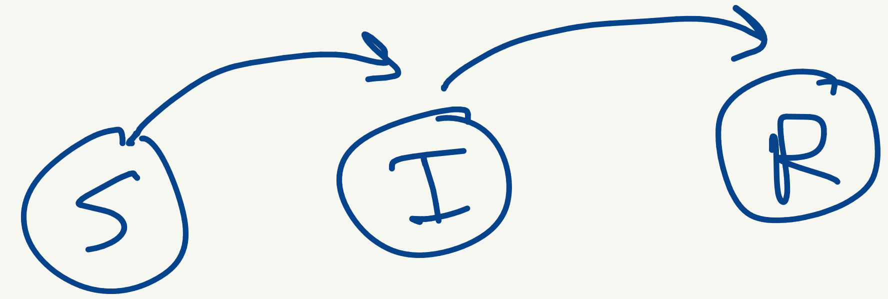
β: transmission rate $\frac{1}{[\text{time}]}$

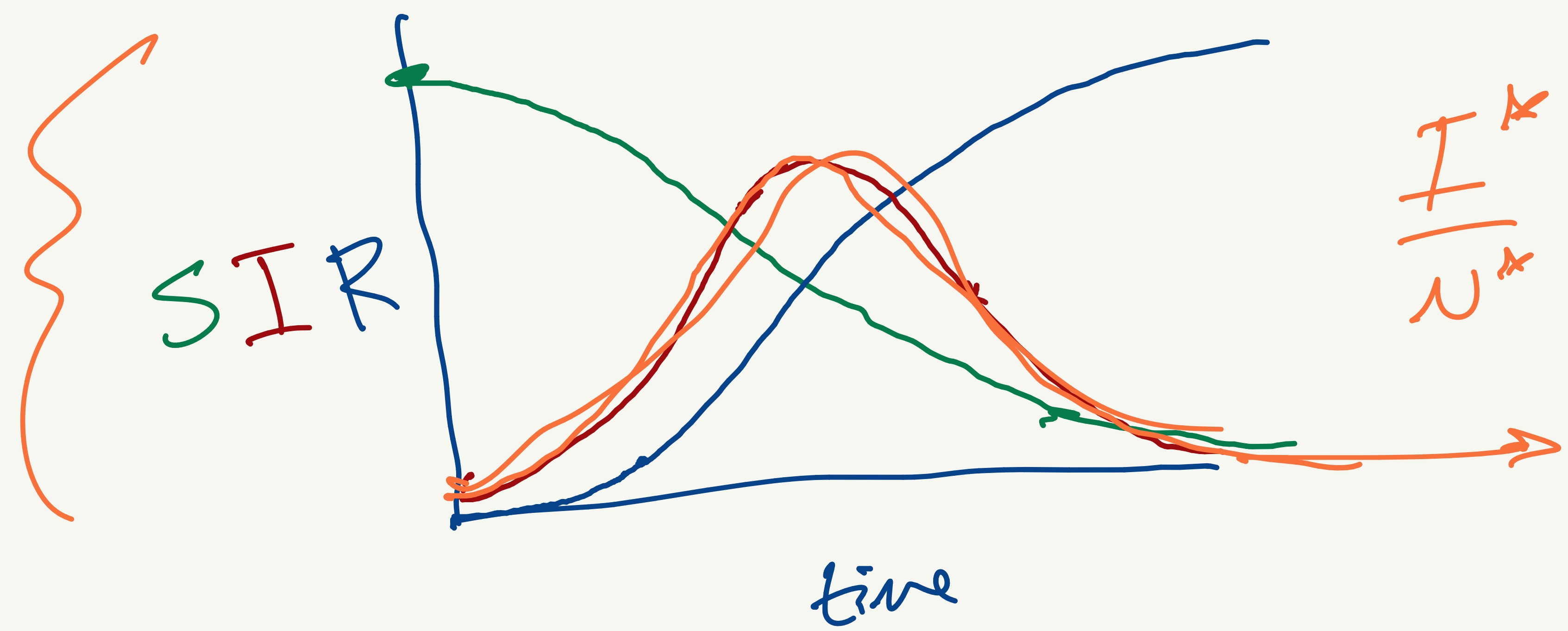
$$\frac{dS}{dt} = -\lambda(I)S = -\beta \frac{I}{N} S$$

$$\frac{dI}{dt} = \lambda(I)S - \gamma I = \beta \frac{I}{N} S - \gamma I$$

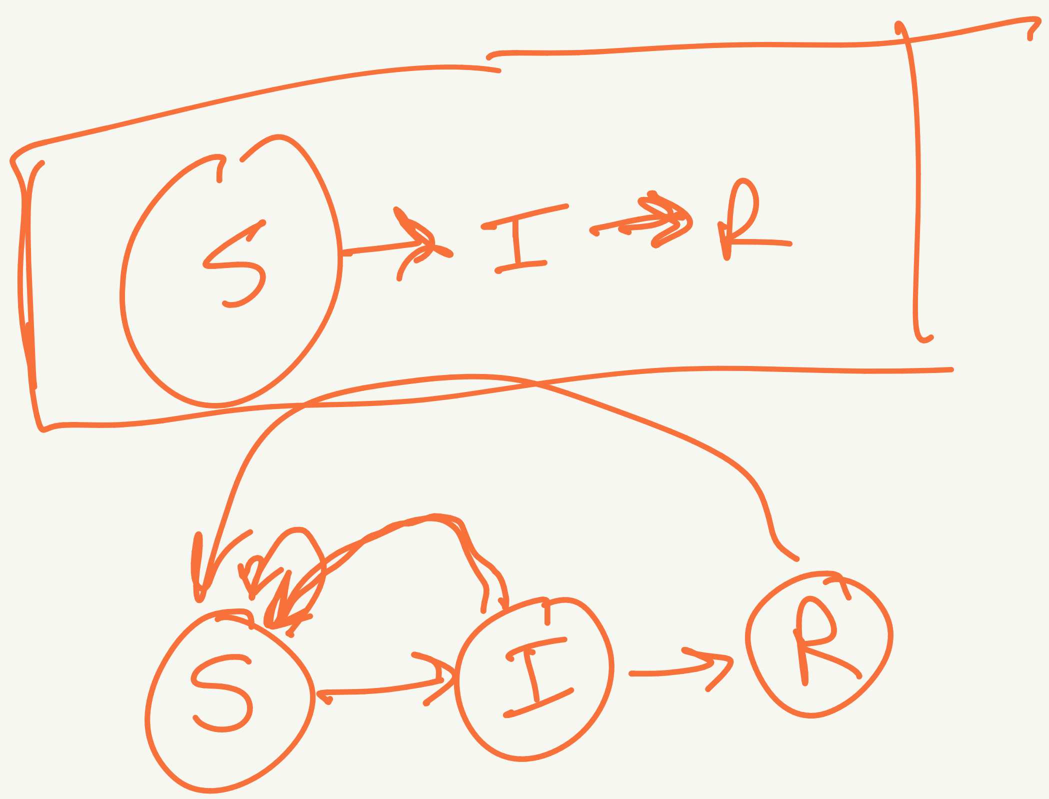
$$\frac{dR}{dt} = \gamma I$$

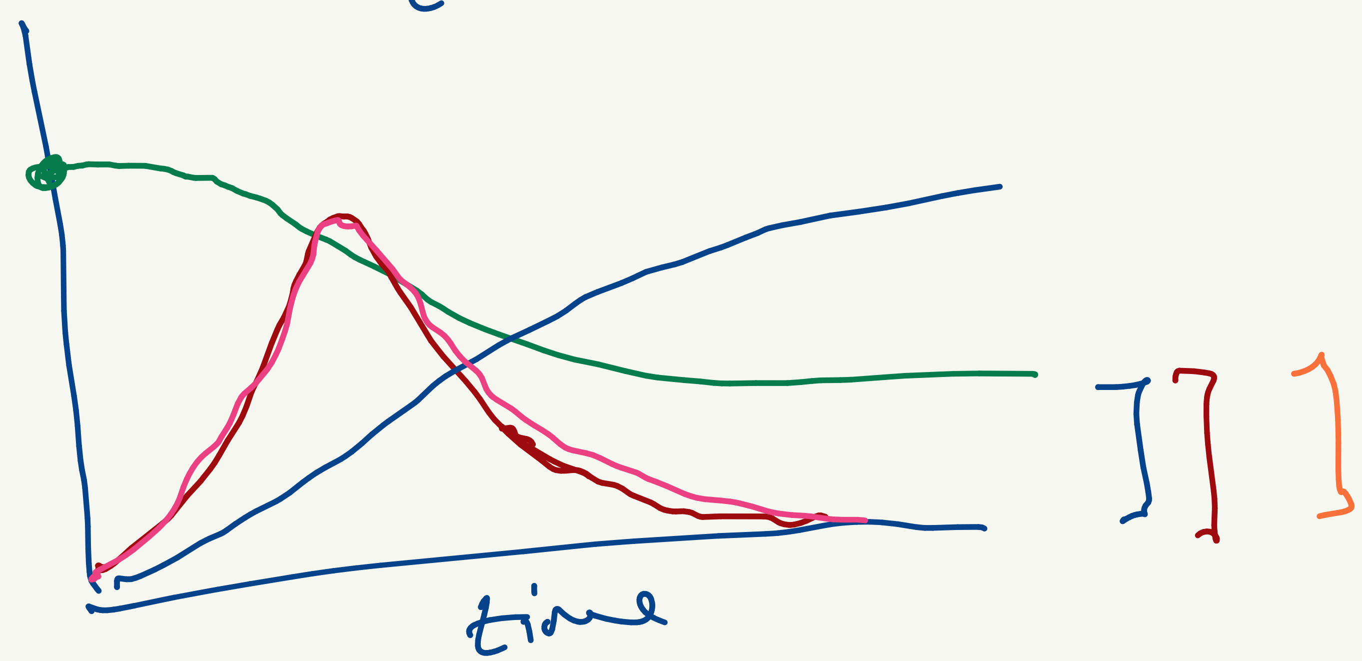
$$\lambda(I) = \beta \frac{I}{N}$$





$$\frac{I^*}{N^*} = \emptyset$$

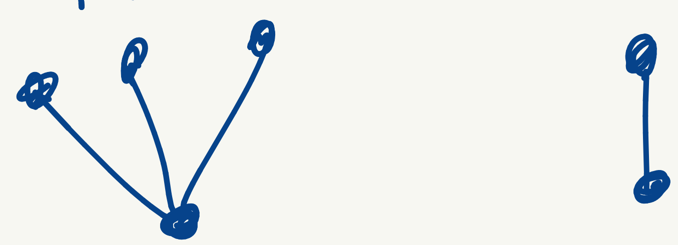




The basic reproductive number R_0

R_0 = the average number of secondary cases arising from a typical primary case in an entirely susceptible population

$R_0 = 3$



$R_0 > 1$ pathogen expected to spread

$\boxed{R_0 < 1}$ pathogen is expected to decline

We assume that the host population @ $t = \emptyset$ has 1 infected individual

$S(\emptyset) = N - 1$

$I(\emptyset) = 1$

The disease will fail to spread if $\frac{dI}{dt} < \emptyset$ at $t = \emptyset$

$$dI/dt < \emptyset$$

$$\beta \frac{I}{N} S - \gamma I < \emptyset$$

$$\beta \frac{\cancel{I}}{N} S < \gamma \cancel{I}$$

transmission rate → β, recovery rate → γ

$$\frac{\beta}{\gamma} \frac{S}{N} < 1$$

$$\frac{\beta}{\gamma} \frac{N-1}{N} < 1$$

$$\frac{\beta}{\gamma} < 1$$

$$R_0 = \frac{\beta}{\gamma}$$

if $\beta/\gamma < 1$ no spread

if $\beta/\gamma > 1$ outbreak

$$R_0 = \left[\frac{\beta}{\gamma}\right] \begin{cases} > 1 & \text{outbreak} \\ < 1 & \text{pathogen dies out} \end{cases}$$

c

$$\frac{dI}{dt} = \beta \frac{I}{N} S - \gamma I - \mu I$$

$\underline{\underline{R_0}}$ <u>Flu</u>: 2-3   HIV/AIDS: 3-5   Smallpox: 5-7   <u>Ebola</u>: 1.5-2.5

<u>Measles</u>: 12-18

$$0 = \beta \frac{I}{N} S - \gamma I - \mu I$$

$$\beta \frac{\cancel{I}}{N} S = \gamma \cancel{I} + \mu \cancel{I}$$

$$\beta \frac{S}{N} = \gamma + \mu \rightarrow \beta \overbrace{\boxed{\frac{N-1}{N}}}^{1} = \gamma + \mu$$

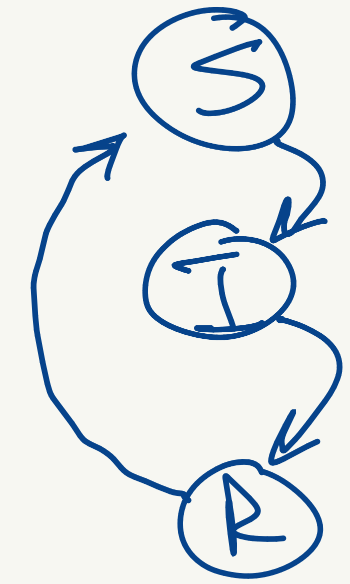
$$R_0 = \frac{\beta}{\boxed{\gamma + \mu}} = \boxed{1}$$

R_0 if we consider < Recovery, Mortality

$$\frac{N-1}{N} \approx 1 \text{ if } N \text{ is large}$$

$$\frac{99999}{100000} = 0.99999 \approx 1$$

Now let's consider the SIR model w/ demographic processes



Assume

1) All individuals can reproduce at rate b
2) All offspring are susceptible
3) All individuals experience mortality at rate μ independent of disease!

$$\frac{dS}{dt} = bN - \beta \frac{I}{N} S - \mu S = b(S+I+R) - \beta \frac{I}{N} S - \mu S$$

$$\frac{dI}{dt} = \beta \frac{I}{N} S - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

R_0?

$$\boxed{\frac{dI}{dt} > 0}$$

$$\beta \frac{\cancel{I}}{N} S > \gamma \cancel{I} + \mu \cancel{I}$$

$$\beta \frac{S}{N} > \gamma + \mu$$

$$\boxed{\frac{\beta}{\gamma + \mu} > 1}$$

$$R_0 = \frac{\beta}{\gamma + \mu}$$

- Small modification to R_0 (accounts for an altered rate <u>out</u> of the infected state: Recovered + Dead)

- Not the full story!

Solve for the steady state! $\underbrace{\frac{dS}{dt} = \emptyset}$, $\underbrace{\frac{dI}{dt} = \emptyset}$, $\underbrace{\frac{dR}{dt} = \emptyset}$

dS/dt

$$\emptyset = bN - \beta \frac{I}{N} S - \mu S$$

$$bN = \beta \frac{I}{N} S + \mu S$$

$$bN = \frac{S}{N}(\beta I + N\mu)$$

$$bN = \left(\frac{\gamma+\mu}{\beta}\right)(\beta I + N\mu)$$

$$\left(\frac{1}{\cancel{N}}\right)\frac{b\cancel{N}\beta}{\gamma+\mu} = (\beta I + \cancel{N}\mu)\left(\frac{1}{\cancel{N}}\right)$$

$$\frac{b\beta}{\gamma+\mu} = \beta\frac{I}{N} + \mu \rightarrow \beta\frac{I}{N} = \frac{b\beta}{(\gamma+\mu)} - \mu \rightarrow \boxed{\frac{I}{N}} = \frac{b}{(\gamma+\mu)} - \frac{\mu}{\beta}$$

Pop. of infected

dI/dt

$$\emptyset = \beta \frac{I}{N} S - \gamma I - \mu I$$

$$\beta \frac{\cancel{I}}{N} S = \gamma \cancel{I} + \mu \cancel{I}$$

$$\beta \frac{S}{N} = \gamma + \mu$$

$$\frac{S}{N} = \frac{\gamma+\mu}{\beta}$$

proportion of susceptibles

$$\emptyset = \gamma I - \mu R$$

$$\gamma I = \mu R$$

$$\left(\frac{1}{N}\right) R = \frac{\gamma}{\mu} I \left(\frac{1}{N}\right)$$

$$\frac{R}{N} = \frac{\gamma}{\mu}\boxed{\frac{I}{N}}$$

$$\frac{R}{N} = \frac{\gamma}{\mu}\left[\frac{b}{(\gamma+\mu)} - \frac{\mu}{\beta}\right]$$

Pop. of recovered

Not zero... not disease-free.

1) This means the disease becomes ENDEMIC $(I^* > \emptyset)$

2) Proportions of $\frac{S}{N}$, $\frac{I}{N}$, $\frac{R}{N}$ at steady state depends on both demographic and disease parameters

What does R_0 mean?

R_0 = the average number of people each sick person infects

$R_0 = 1$: $O \to O$

$R_0 = 2$: $O \to O$, $O \to O$

$R_0 = <1$: $O \to O$

$0.5 \to O$

$R_0 > 1$ epidemic continues

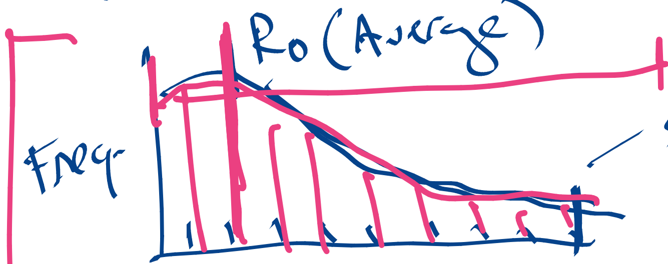
exponential growth

$R_0 < 1$ epidemic decline

What does R_0 mean?

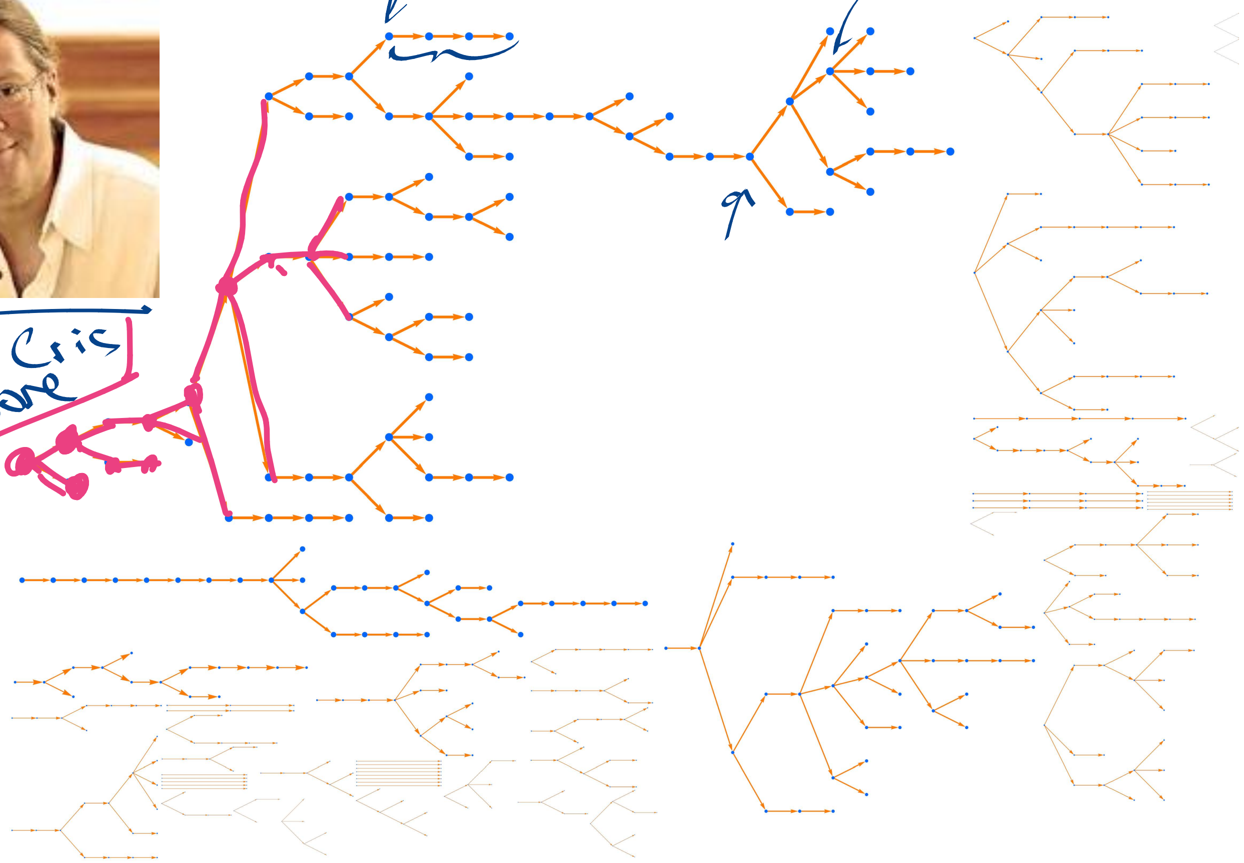
- Virus life cycle
- Human physiology
- human social dynamics

R_0 ~ describes the AVERAGE



# of people each infected person infects

~~Chris~~ Cris Moore



$\boxed{R_0 < 1}$

Superspreaders

A hundred random outbreaks in a scenario where each sick person interacts with 10 others, and infects each one with probability 8 percent. Here R0 = 0.8 and the average outbreak size is five, but 1 percent of the outbreaks have size 50 or larger, and in this run the largest has size 82.

What does R_0 mean?

$R_0 = 0.8$

A hundred random outbreaks in a scenario with superspreading, where 1 percent of the cases infect 20 others. As in Figure 1, we have R0 = 0.8 and the average outbreak size is 5, but now the heavy tail of outbreaks is much heavier. In this run the largest outbreak has size 663.