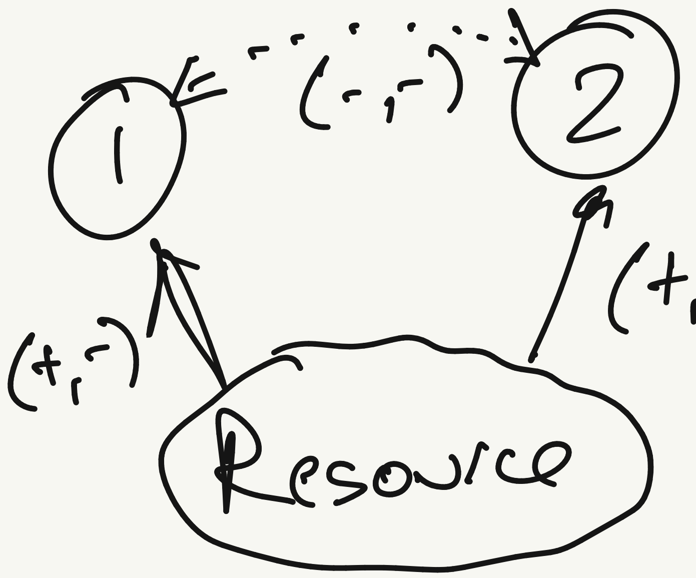
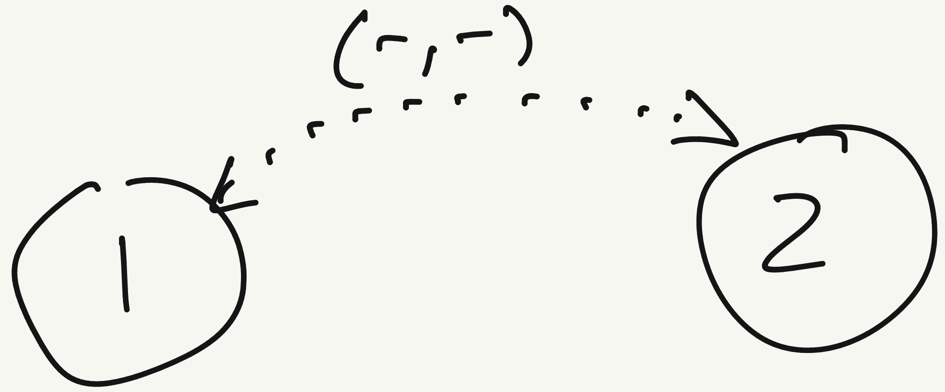
Lotka-Volterra Competition System

(System of equations)





2D-L-V Competition System

$$\begin{cases} \dfrac{dN_1}{dt} = r_1 N_1 \left(1 - \dfrac{N_1 + \alpha N_2}{K_1}\right) \\[2ex] \dfrac{dN_2}{dt} = r_2 N_2 \left(1 - \dfrac{N_2 + \beta N_1}{K_2}\right) \end{cases}$$

α = per-capita effect of species 2 on the growth of species 1

β = per-capita effect of species 1 on the growth of species 2

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha N_2}{K_1}\right)$$

Increased α means increased negative effect of N_2 on N_1

$$\overbrace{r_1 N_1^* \left(1 - \frac{N_1^* + \alpha N_2}{K_1}\right)} = \emptyset$$

assuming $r_1 > \emptyset$

Solution 1: $N_1^* = \emptyset$

Solution 2: $1 - \overbrace{\frac{N_1^* + \alpha N_2}{K_1}} = \emptyset$

$$1 = \frac{N_1^* + \alpha N_2}{K_1}$$

$$K_1 = N_1^* + \alpha N_2$$

$$\underbrace{N_1^* = K_1 - \alpha N_2}$$

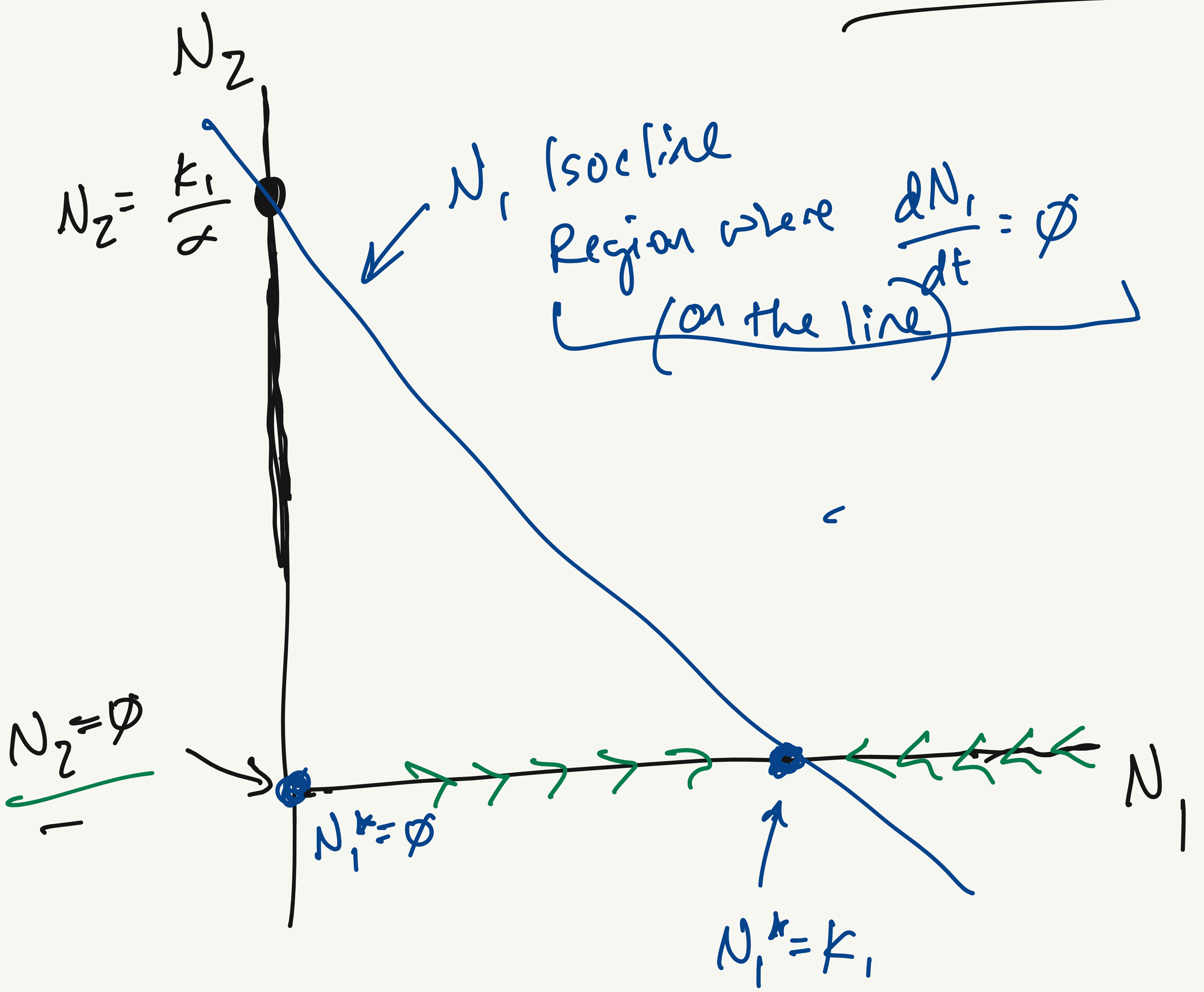
Under the condition $N_2 = \emptyset$

$$\boxed{N_1^* = K_1 - \alpha(\emptyset)}$$

$$N_1^* = K_1$$

→ $N_1^* = \underbrace{K_1 - \alpha N_2}$ Zero net growth isocline (ZNGI)

⊛ N_1 Isocline



$N_1^* = K_1 - \alpha N_2$

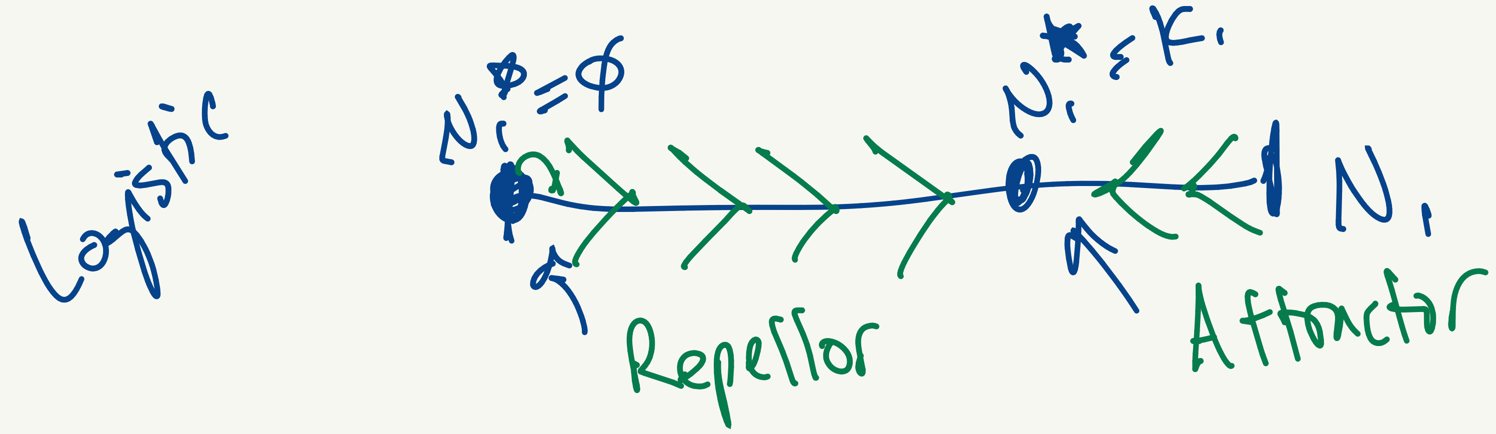
x-intercept → $N_1^* = K_1$
@ $N_2 = \emptyset$

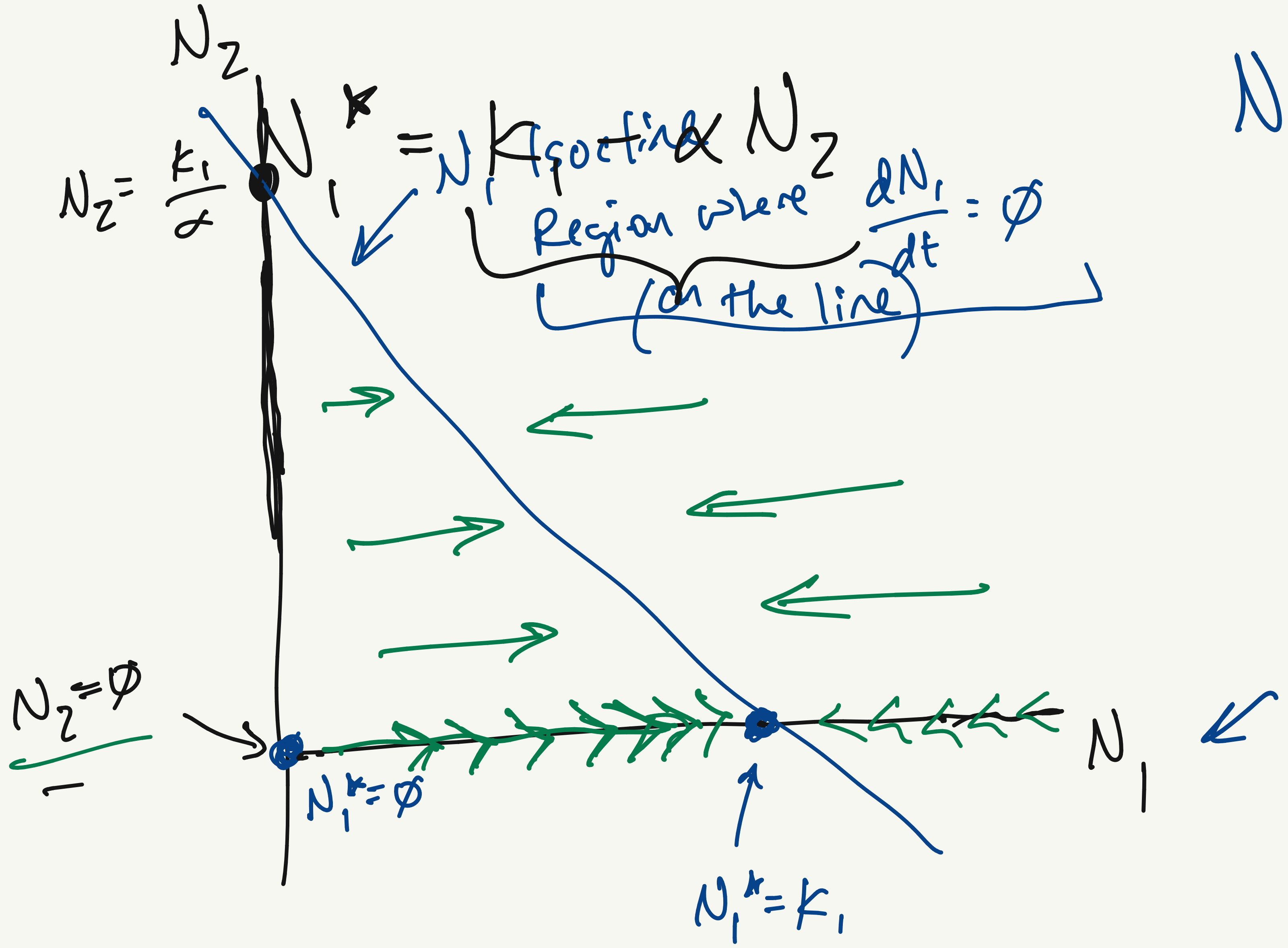
y-intercept → $\emptyset = K_1 - \alpha N_2$
@ $N_1 = \emptyset$

$\alpha N_2 = K_1$

$N_2 = \frac{K_1}{\alpha}$

N_2	Large N_2 Small N_1	Large N_2 Large N_1
	Small N_2 Small N_1	Small N_2 Large N_1
	N_1	





Now we want to describe FLOW on either side of the isocline

$$\frac{dN_1}{dt} > 0 \ ?$$

$$\underbrace{r_1 N_1}_{(+)} \left(1 - \frac{N_1 + \alpha N_2}{K_1}\right) > 0$$

$$1 - \frac{N_1 + \alpha N_2}{K_1} > 0$$

$$1 > \frac{N_1 + \alpha N_2}{K_1}$$

$$K_1 > N_1 + \alpha N_2$$

SAME $\begin{cases} K_1 - \alpha N_2 > N_1 \\ N_1 < K_1 - \alpha N_2 \end{cases}$

Growth if $N_1 < \underbrace{K_1 - \alpha N_2}_{N_1 \text{ Isocline!}}$

When is $\frac{dN_1}{dt} < 0$

$$N_1 > K_1 - \alpha N_2$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \beta N_1}{K_2}\right)$$

Solve for Steady state

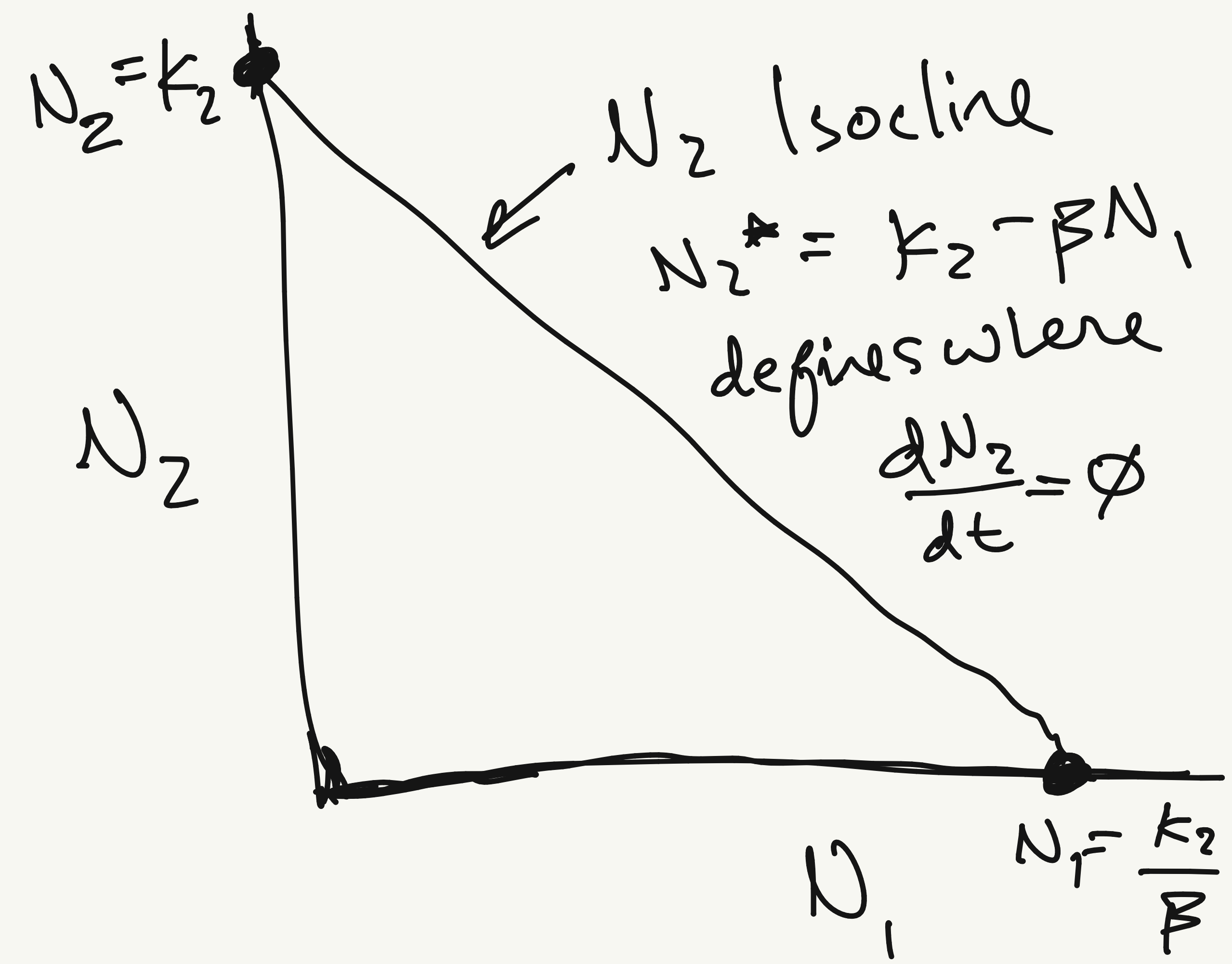
$$\frac{dN_2}{dt} = \emptyset \qquad \underbrace{r_2 N_2}\underbrace{\left(1 - \frac{N_2^* + \beta N_1}{K_2}\right)} = \underline{\underline{\emptyset}}$$

$$1 - \frac{N_2^* + \beta N_1}{K_2} = \emptyset$$

$$1 = \frac{N_2^* + \beta N_1}{K_2}$$

$$K_2 = N_2^* + \beta N_1$$

$$\left[\; K_2 - \beta N_1 = N_2^* \;\right] \; \underline{\underline{N_2 \text{ Isocline}}}$$



Solve for Intercepts

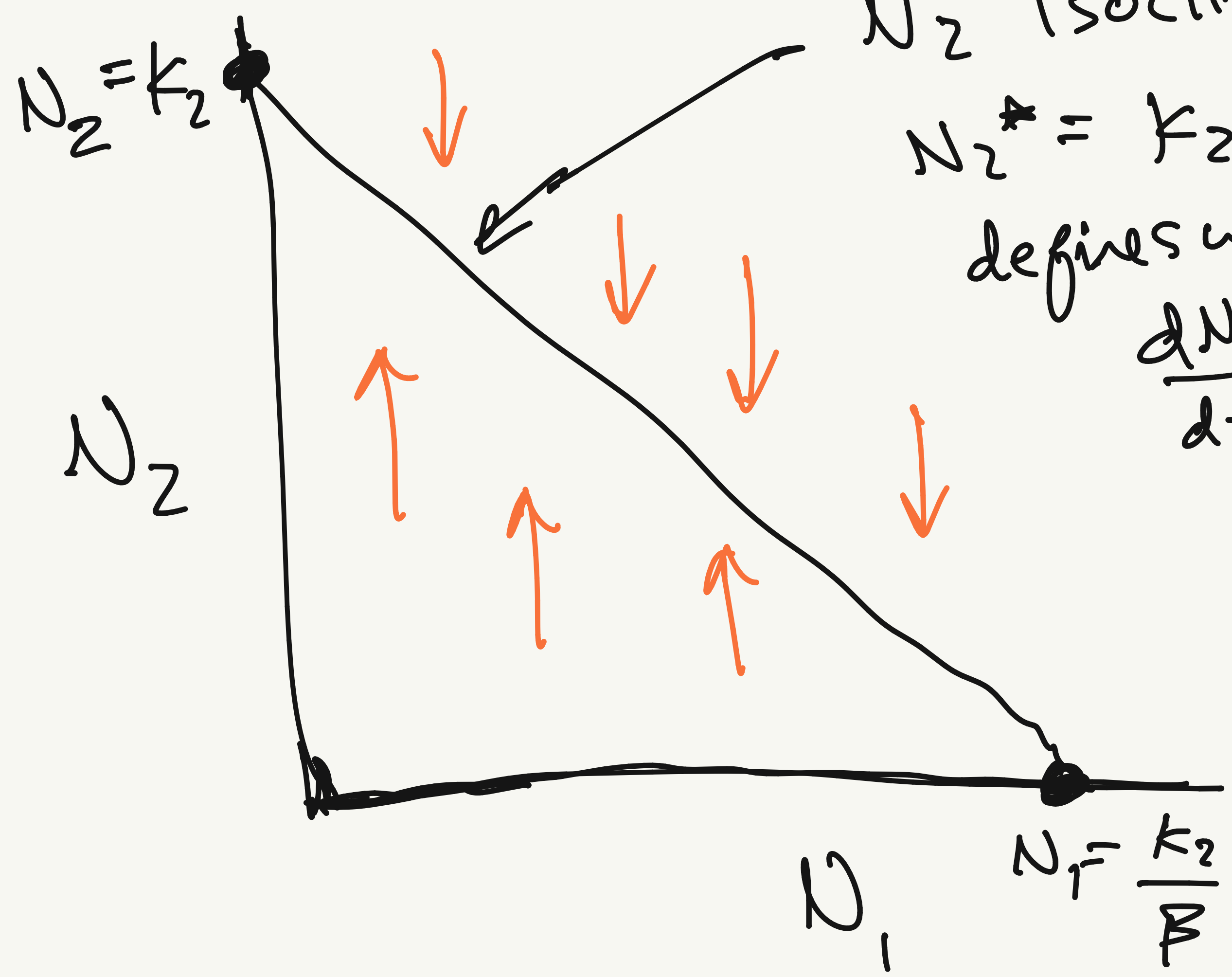
y-intercept @ $N_1 = \emptyset$ $\rightarrow$ $K_2 - \beta(\emptyset) = N_2^*$

$$N_2^* = K_2$$

x-intercept @ $N_2 = \emptyset$ $\rightarrow$ $K_2 - \beta N_1 = \emptyset$

$$K_2 = \beta N_1$$

$$N_1 = \frac{K_2}{\beta}$$



N_2 Isocline

$N_2^* = K_2 - \beta N_1$

defines where

$\frac{dN_2}{dt} = \emptyset$

Decline $\frac{dN_2}{dt} < \emptyset$

$N_2 > K_2 - \beta N_1$

When does N_2 grow?

$\frac{dN_2}{dt} > \emptyset$?

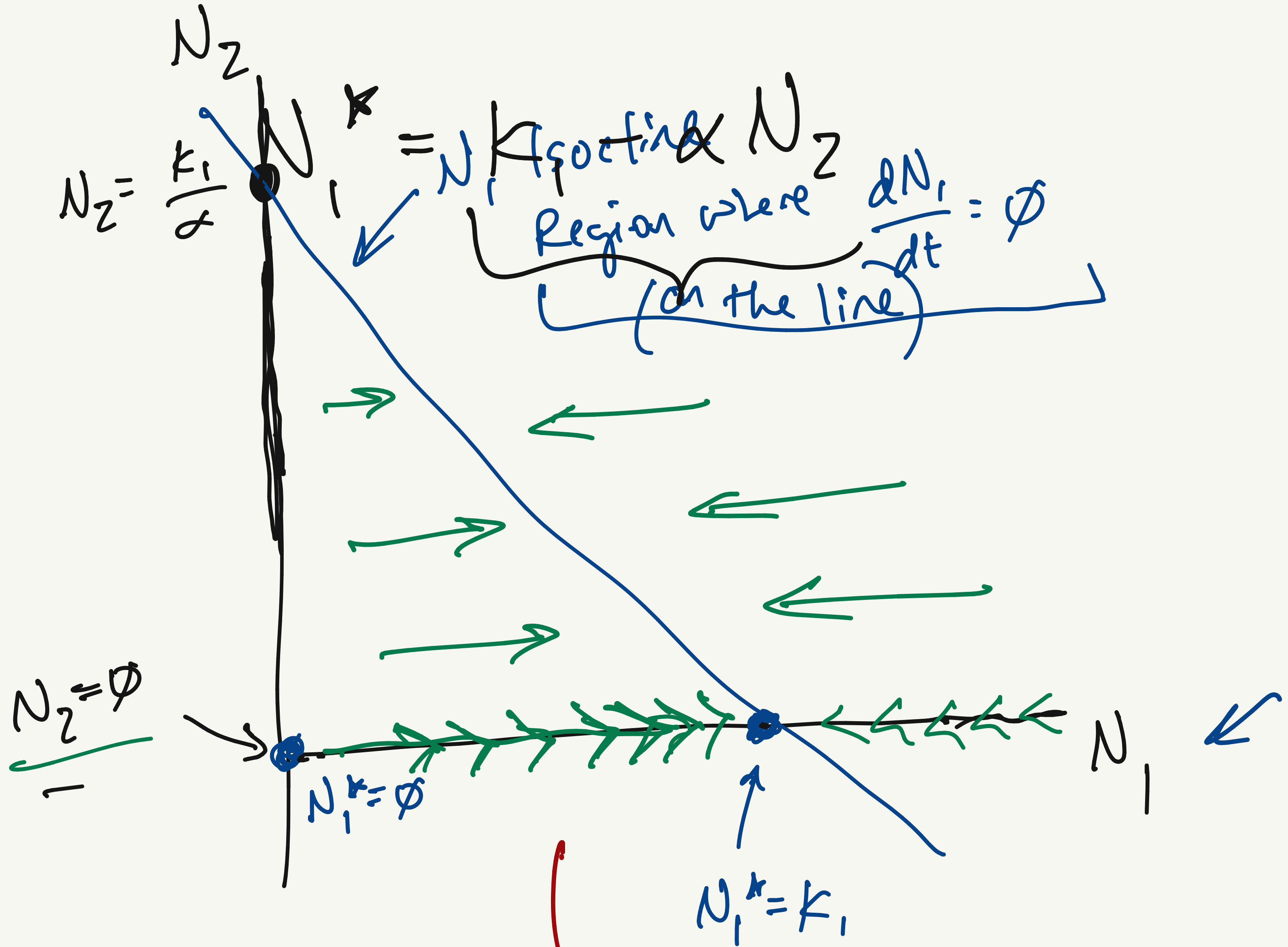
$$\underbrace{r_2 N_2}_{> \emptyset} \underbrace{\left(1 - \frac{N_2 + \beta N_1}{K_2}\right)}_{} > \emptyset$$

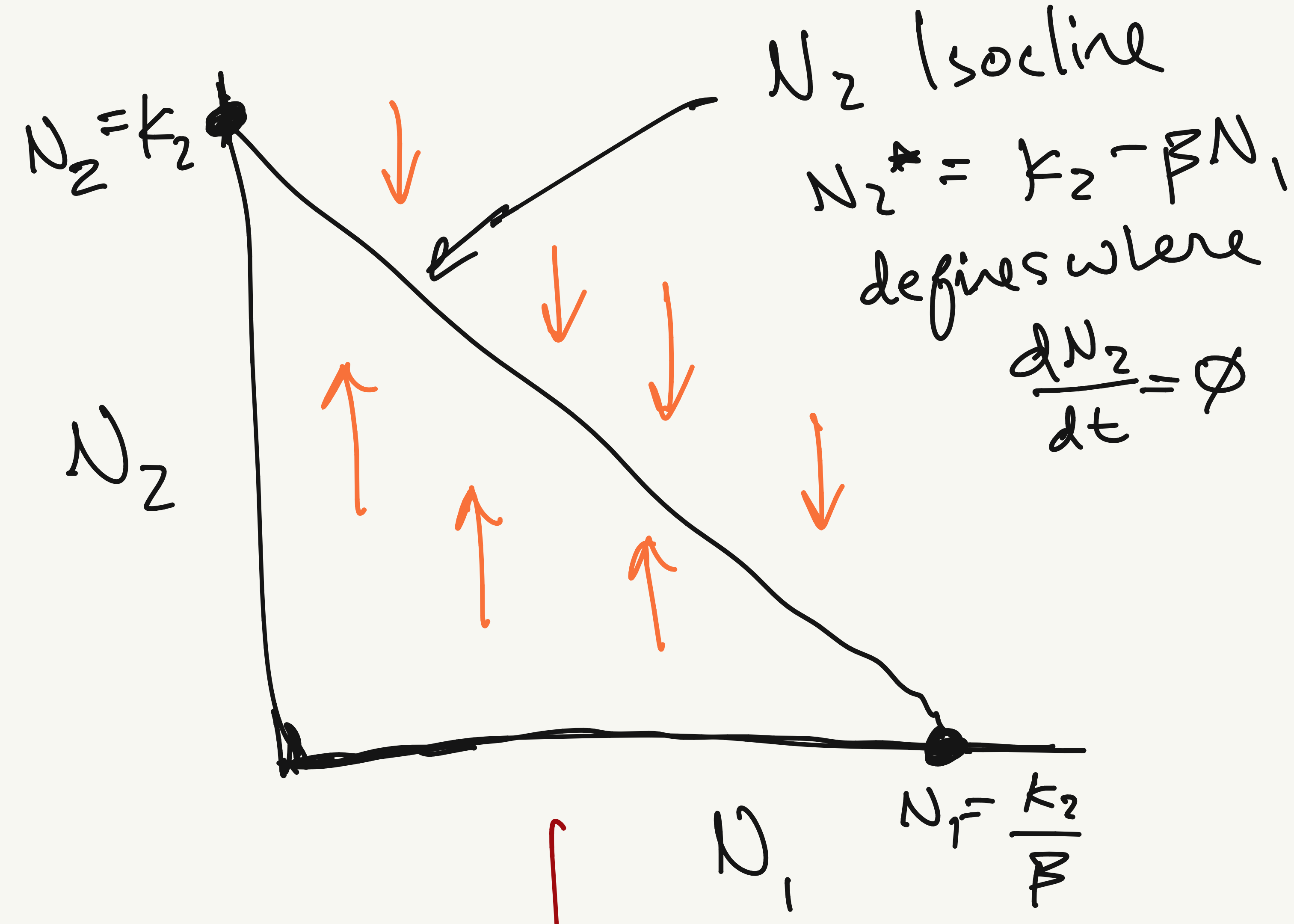
$$1 - \frac{N_2 + \beta N_1}{\underbrace{K_2}_{}} > \emptyset$$

$$1 > \frac{N_2 + \beta N_1}{K_2}$$

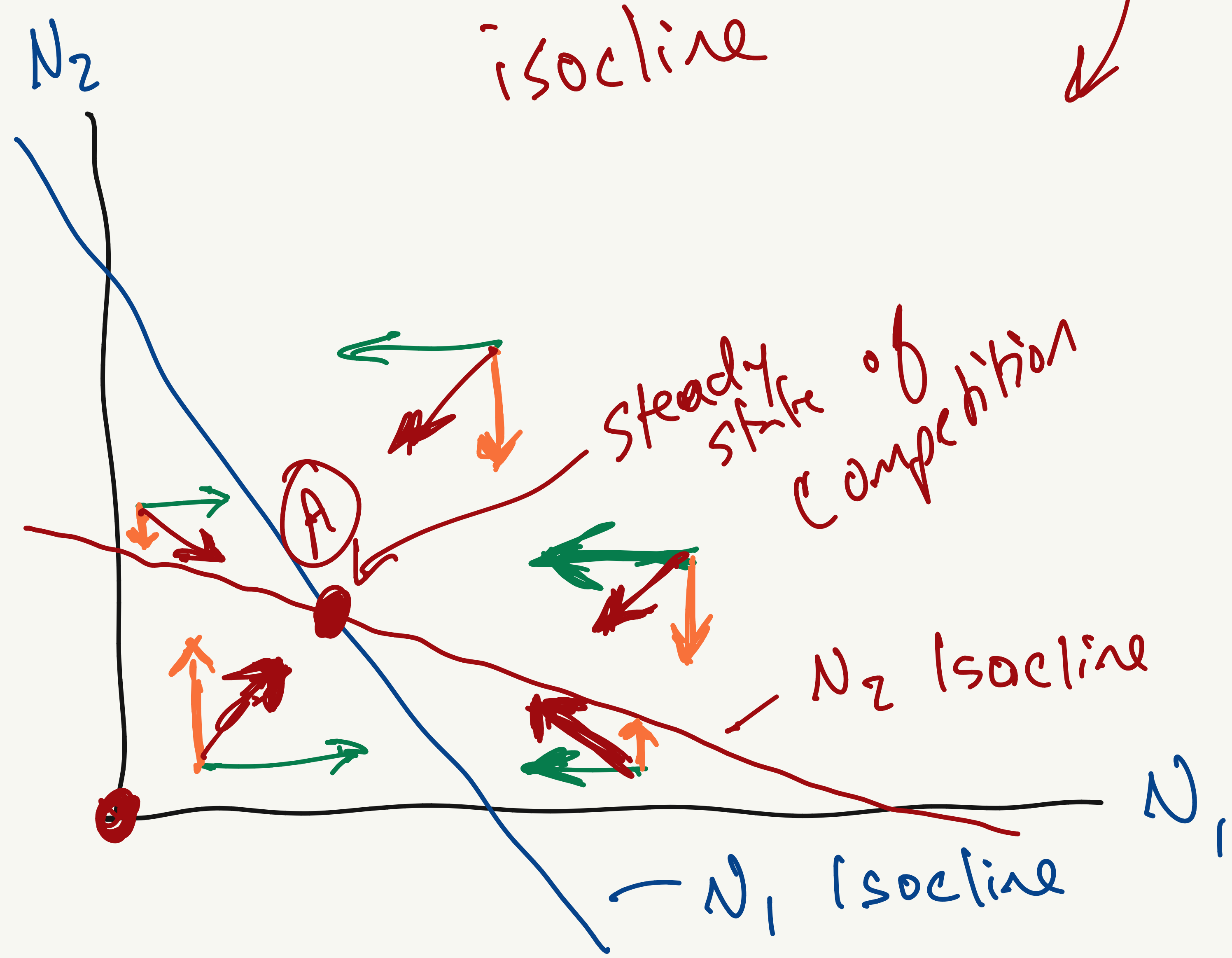
$$K_2 > N_2 + \beta N_1$$

SAME $\begin{cases} K_2 - \beta N_1 > N_2 \\ N_2 < \underbrace{K_2 - \beta N_1}_{} \end{cases}$ ← N_2 Isocline!





isocline



A is only place when both $\frac{dN_1}{dt} = \varnothing$, $\frac{dN_2}{dt} = \varnothing$

$\begin{pmatrix} K_1, K_2 \\ \alpha, \beta \end{pmatrix}$ determine orientation of N_1, N_2 isoclines