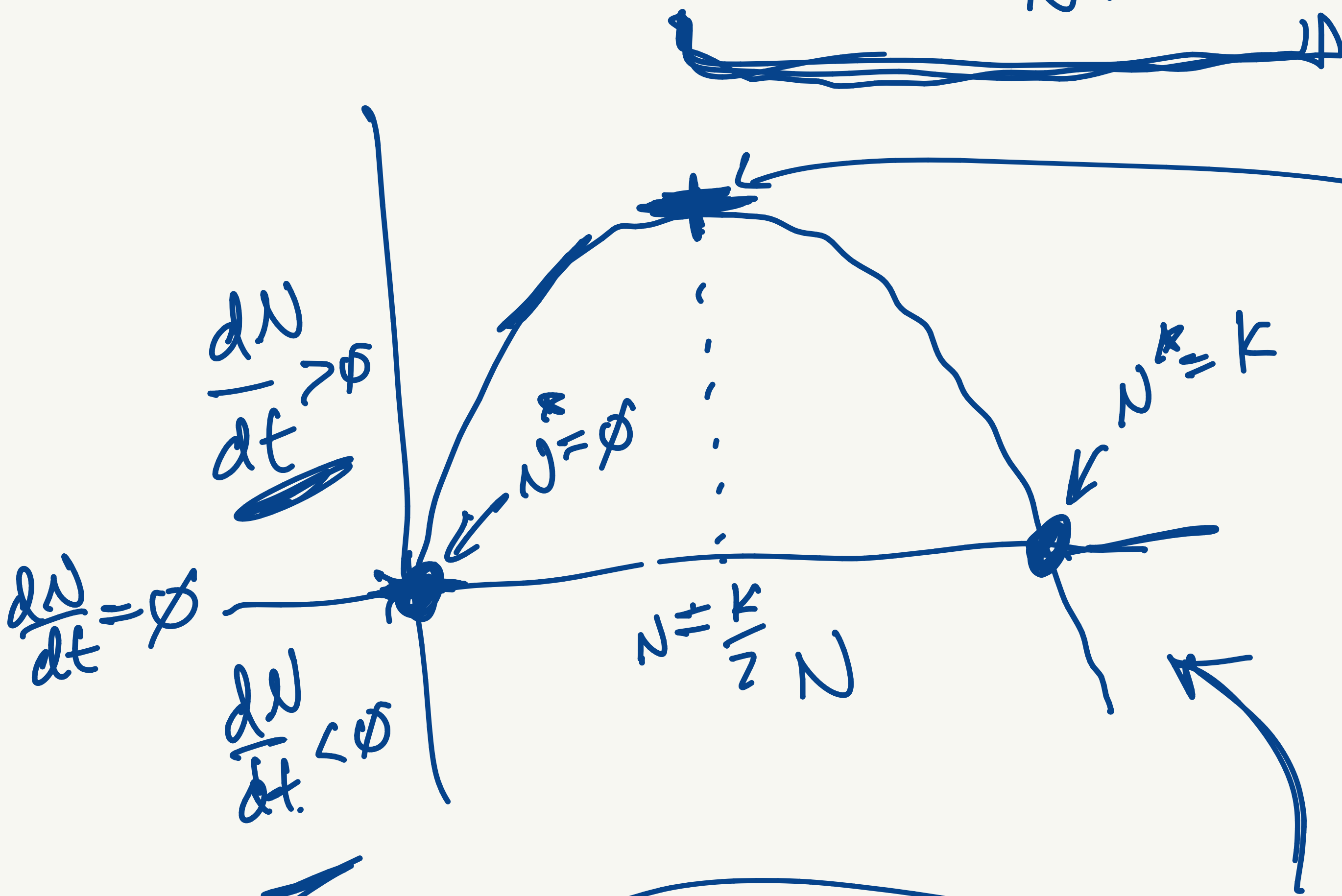
Continuous-time Logistic equation

$$\frac{dN}{dt} = rN\underbrace{\left[1 - \frac{N}{K}\right]}_{N \approx \emptyset}$$

1 ~ exponential growth (nearly)

$\underbrace{\quad}_{N \approx K}$ $\emptyset$ $\frac{dN}{dt} \approx \emptyset$ steady state

Solve for max $\frac{dN}{dt}$



take derivative $rN\left[1 - \frac{N}{K}\right]$ & set to zero

$$rN - \frac{rN^2}{K}$$

Derivative: $r - \frac{2rN}{K} = \emptyset$

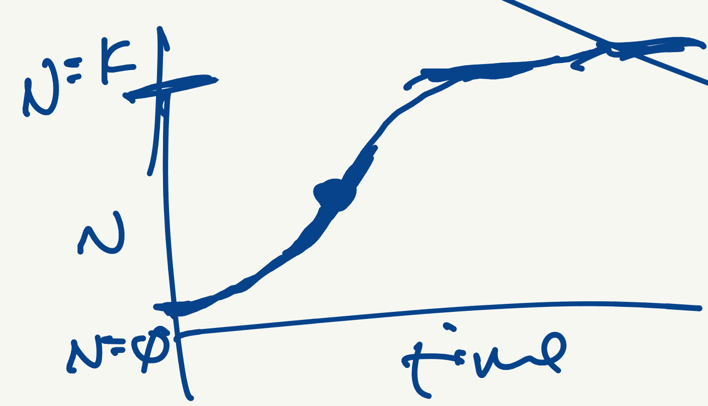
$$\cancel{r}\left(1 - \frac{2N}{K}\right) = \frac{\emptyset}{r}$$

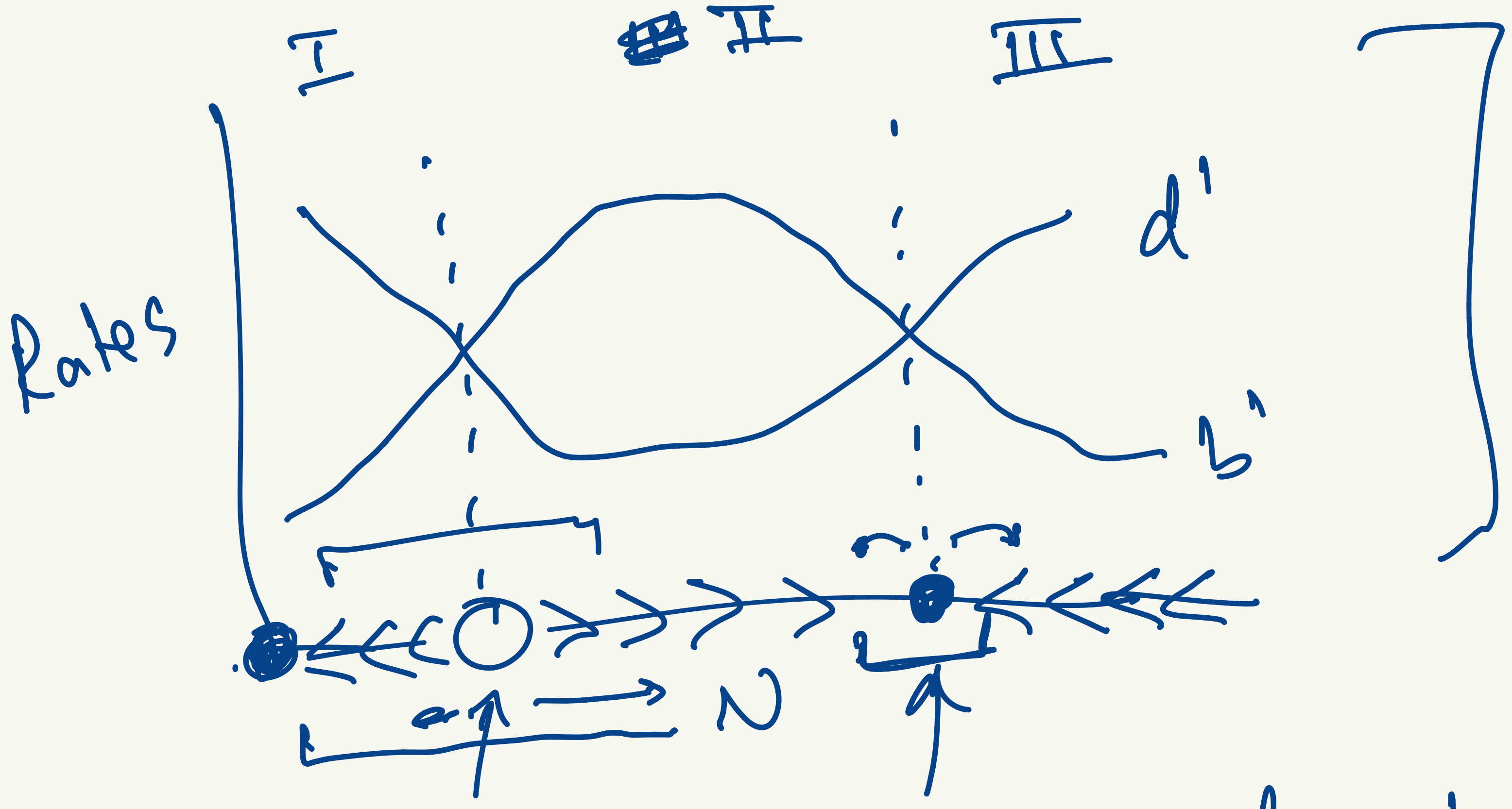
$$1 - \frac{2N}{K} = \emptyset$$

$$\frac{2N}{K} = 1$$

$$2N = K$$

$$\boxed{N = \frac{K}{2}}$$





Unstable steady state

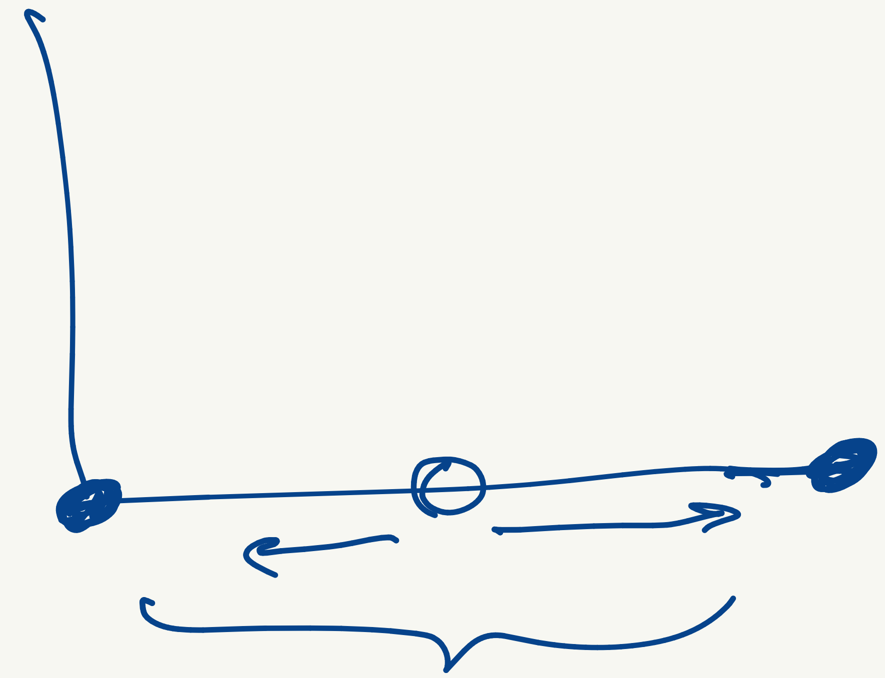
(Repellor)

Stable steady state

(Attractor)

Allee Effect

Critical population size where extinction is inevitable



$$\frac{dN}{dt} = rN\left(1 - \frac{N\uparrow}{K}\right)$$

gets closer to 1

so $\left(1 - \frac{N}{K}\right)$ gets closer to zero

so $\frac{dN}{dt}$ gets closer to zero

Every individual added to the population is slowing down the growth

- We can add other negative effects into the numeractor
- Let's consider a competitor with population size C

if the population of $N\uparrow$, then pop. growth of N should decline

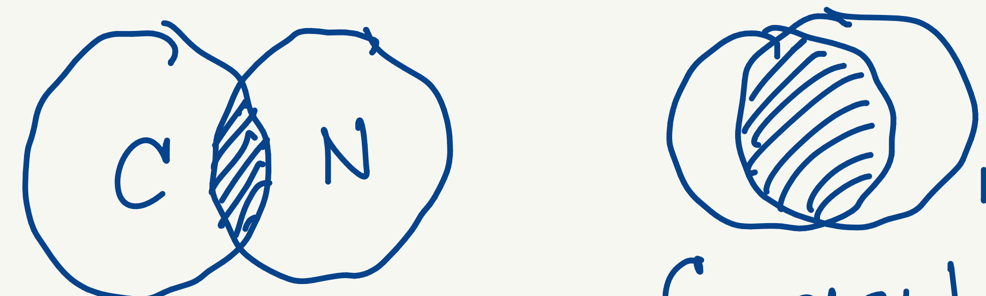
if the population size of $C\uparrow$, then pop. growth of N should decline

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right)$$

$$\frac{dN}{dt} = rN\left(1-\frac{N+\alpha C}{K}\right) \qquad \alpha = 1$$

Where α tells us how "much" of a competitor C is. If $\alpha = 1$, C competes equally as individuals in N (N & C eat the same things)

If $\alpha < 1$, then C doesn't eat quite the same things as N and the effect of competition is <u>less</u>



low α $\qquad$ C $\alpha \approx 1$ N

Where is $\frac{dN}{dt} = 0$?

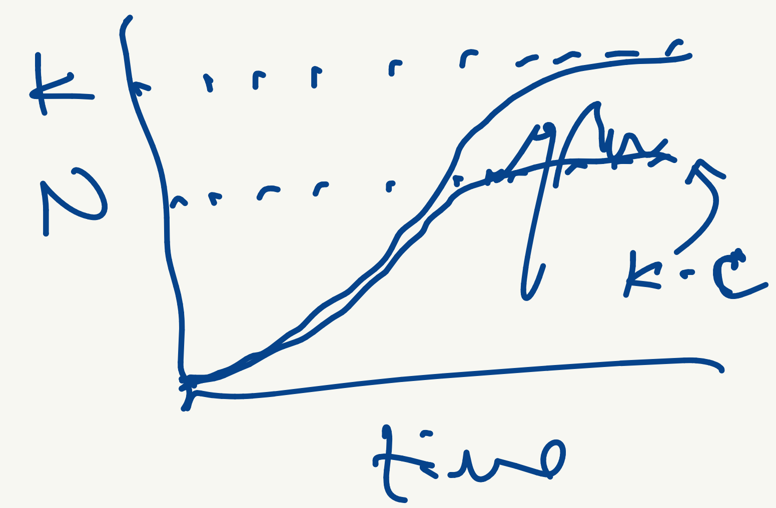
$$rN^*\left(1-\frac{N^*+\alpha C}{K}\right) = 0$$

solve for N^*

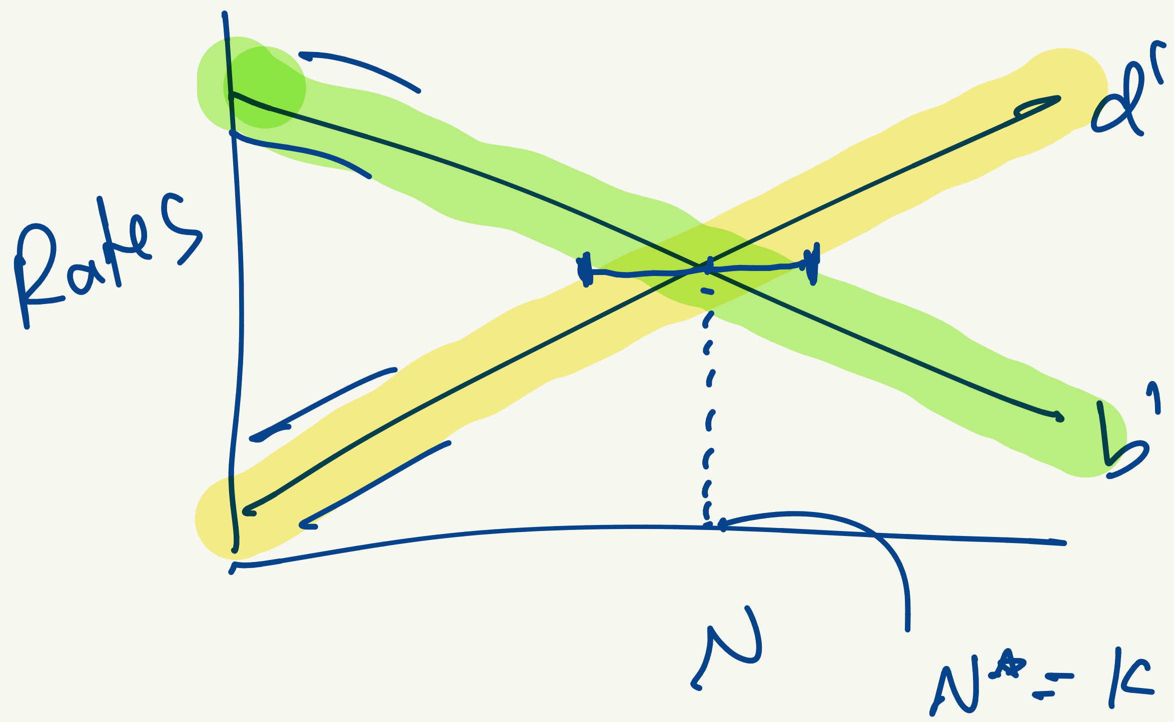
$$N^* = K - \alpha C$$

$\alpha \approx 0 \quad N^* \approx K$

$\alpha \approx 1 \quad N^* \approx K - C$



Fluctuations

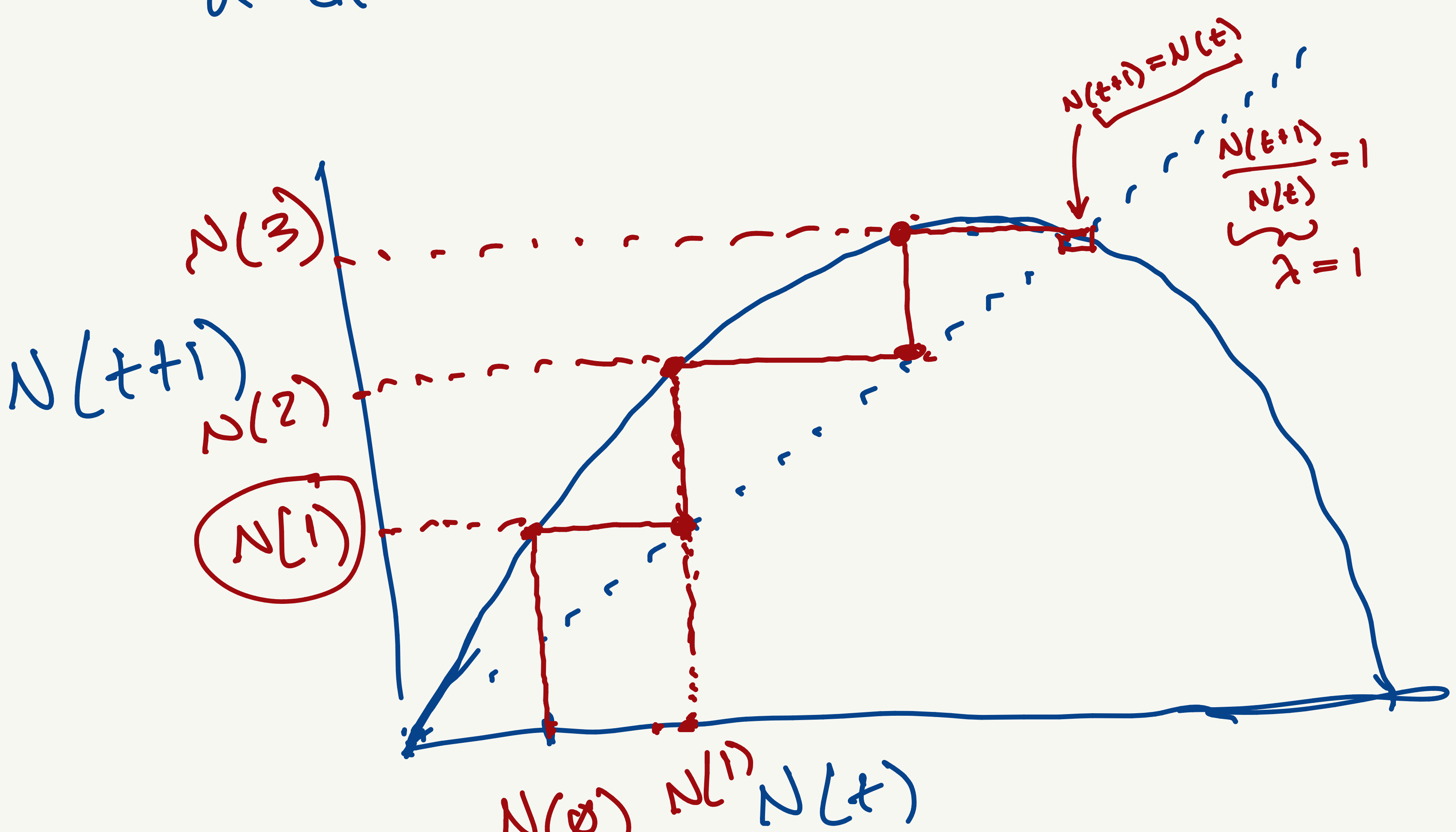


$$N(t+1) - N(t) = r_d N(t)$$

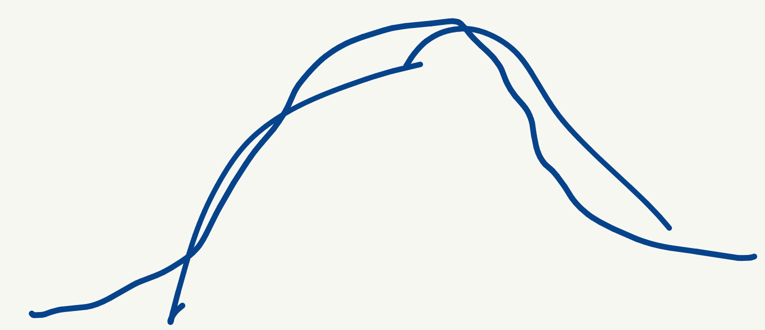
$$N(t+1) - N(t) = r_d N(t)\left(1 - \frac{N(t)}{K}\right)$$ Discrete-time logistic growth

COBWEB Diagram allows us to track the dynamics of a ~~discrete~~ discrete time system

$$N(t+1) = \underbrace{N(t) + r_d N(t)\left(1 - \frac{N(t)}{K}\right)}_{\text{parabola}}$$



you have $N(\emptyset)$



$$N(t+1) = N(t) + r\left(1 - \frac{N(t)}{K}\right) \text{ where } r = 1, \; K = 1$$

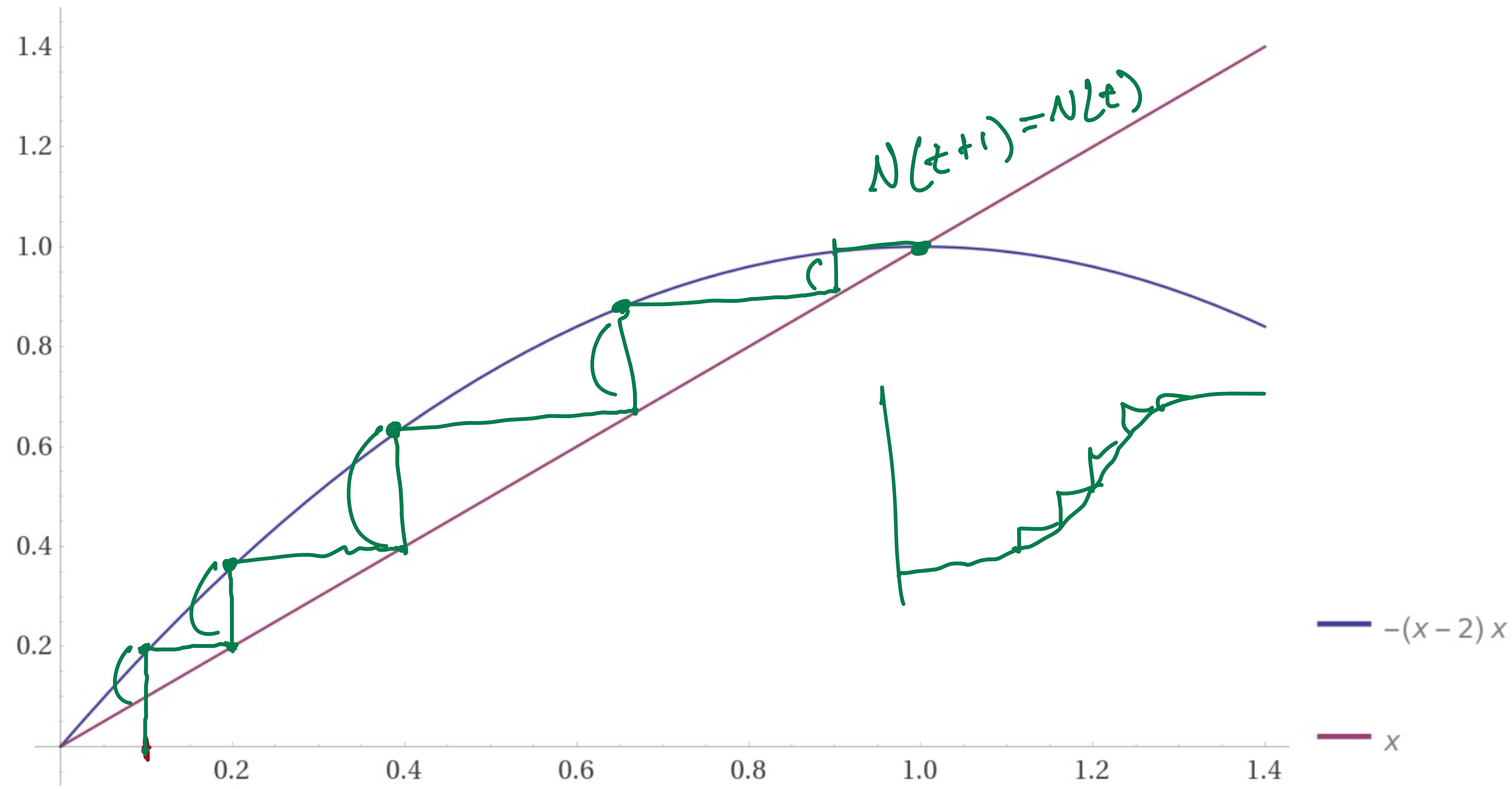
$\frac{\text{inds}}{\text{area}}$

$0.1 \frac{\text{inds}}{\text{km}^2}$

Input interpretation

plot	$x + x(1 - x)$ x	$x = 0$ to 1.4

Plot



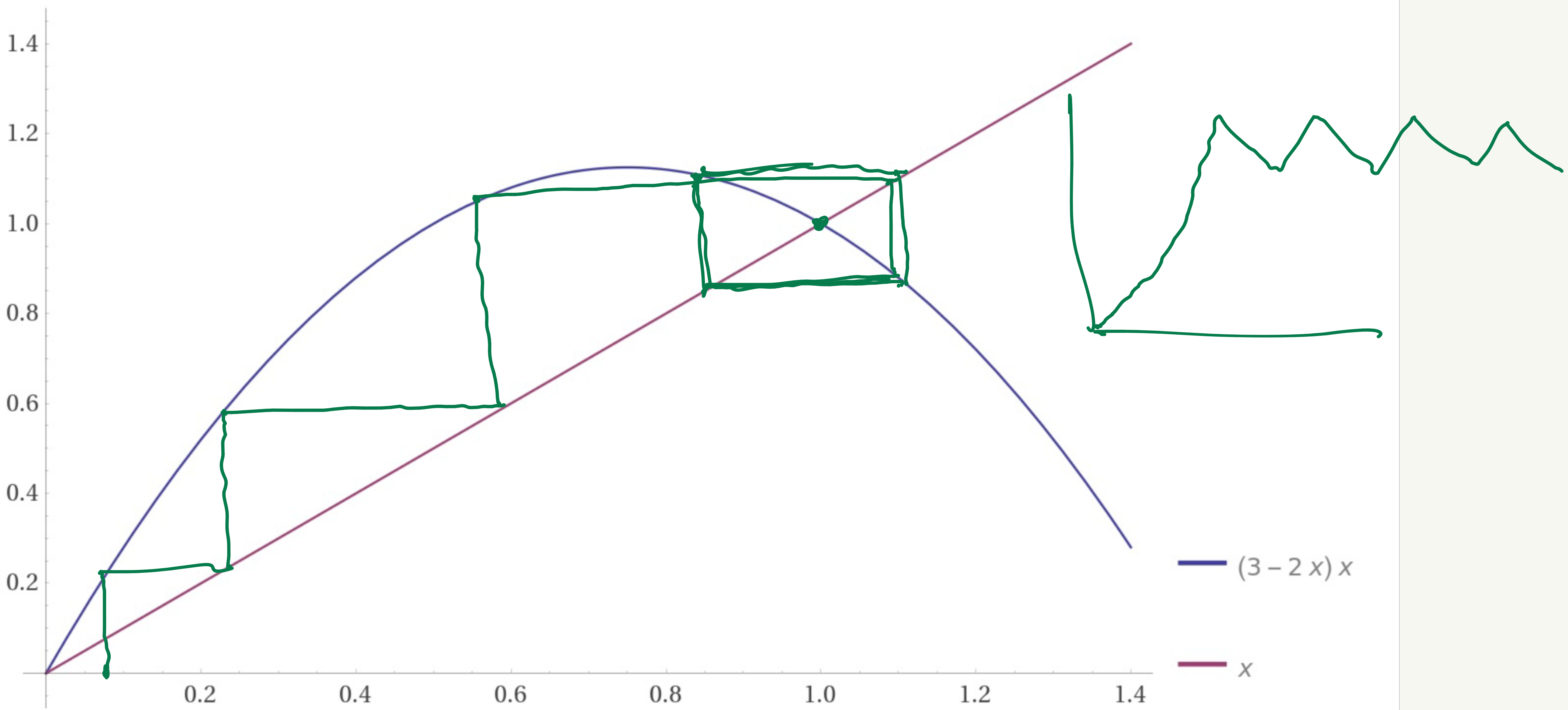
Parametric plot

$r = 2$

Input interpretation

plot	$x + 2x(1-x)$ x	$x = 0$ to 1.4

Plot



chaos

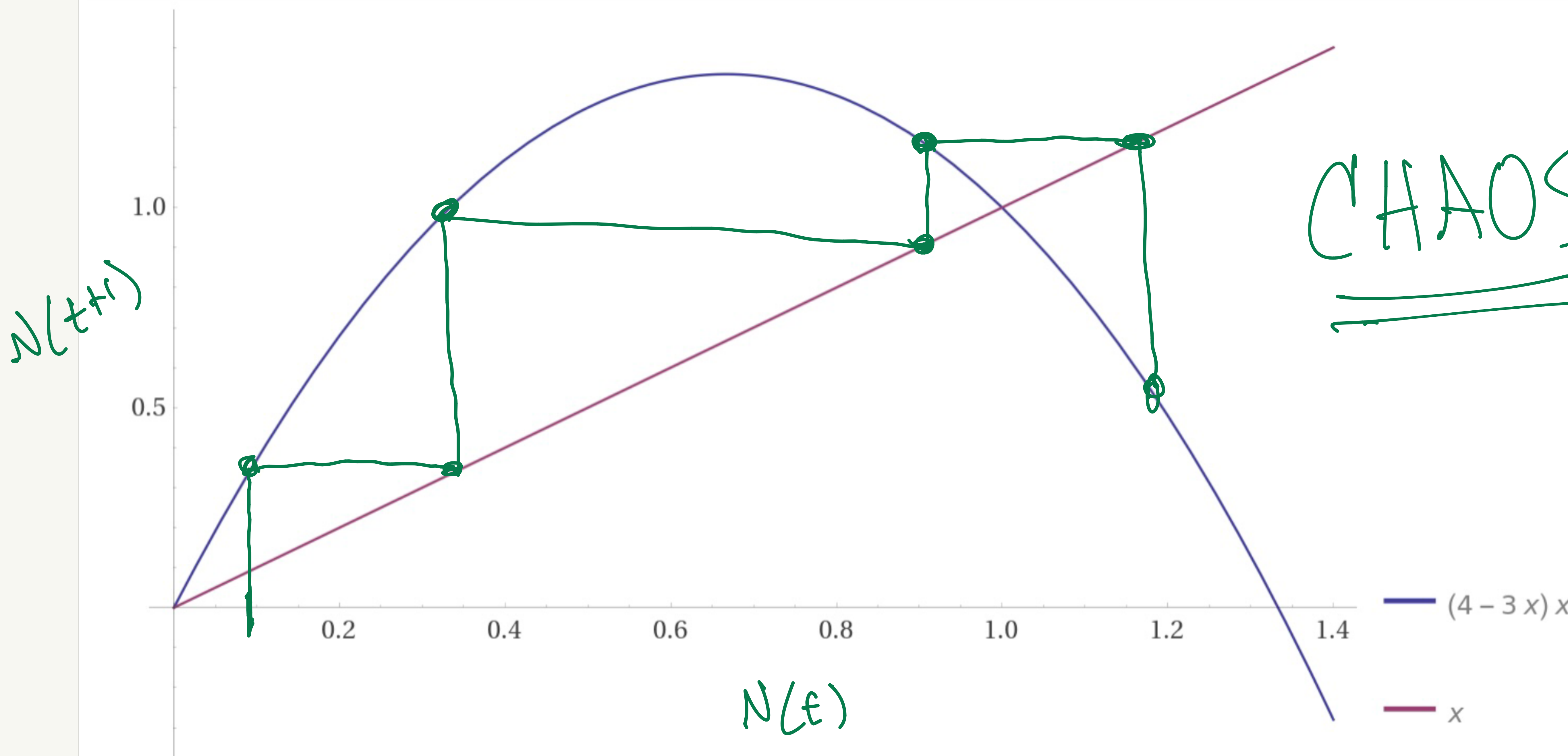
Input interpretation

$r = 3$

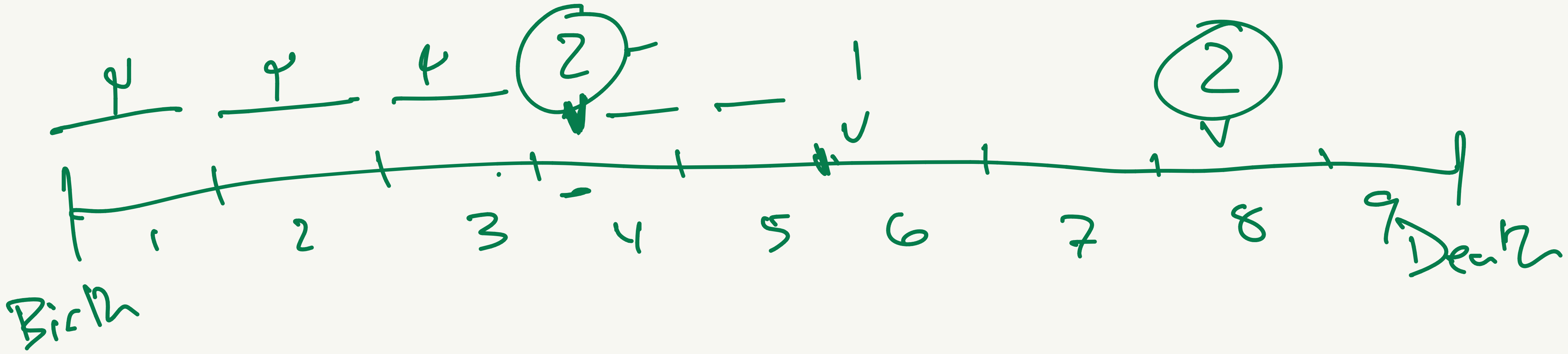
plot	$x + 3x(1 - x)$	$x = 0$ to 1.4
	x	

(Problematic)

Plot



CHAOS



Survival
Reproduction

Nonzero Fitness

1. Survive intervals 1–3 give birth 2 die

2. Survive 1–3 (+2 offspring) Survive 4–5 (+1 offspring)

   → Survive 1–5 (+3)

3) Survive 1–3 (+2), survive 4–5 (+1) OR Survive 6–7 (+2)

   → survive 7 intervals +5 offspring

Surviving 1 interval $1-\mu$

2 intervals $(1-\mu)^2$

3 intervals $(1-\mu)^3$

Offspring is Number offspring $\times (1-\mu_0)$

1) $(1-\mu)^3(2)(1-\mu_0)$ OR

2) $(1-\mu)^5(3)(1-\mu_0)$

3) $(1-\mu)^7(5)(1-\mu_0)$

$$\Phi = (1-\mu)^3(2)(1-\mu_0) + (1-\mu)^5(3)(1-\mu_0) + (1-\mu)^7(5)(1-\mu_0)$$

Decision-Making and Fitness

Φ - future fitness of organism

$$\begin{cases} \text{Yes} & \Phi' = (1+a)\phi + (1-c)\underbrace{(\Phi - \phi)}_{\text{close to zero}} \\ \text{No} & \Phi'' = (1-b)\phi + (\Phi - \phi) \end{cases}$$

$(1+a)\phi > (1-b)\phi$

then **YES**

$(1+a)\phi < (1-b)\phi$

then **NO**

ϕ is amt at stake

- a ~ proportional gain in ϕ in yes decision
- c ~ cost $\emptyset$
- b ~ amt ~~at risk~~ lost if no decision

$\Phi' > \Phi''$ the fitness assoc. w/ a 'yes' decision is higher

$\Phi' < \Phi''$ No decision is more fit