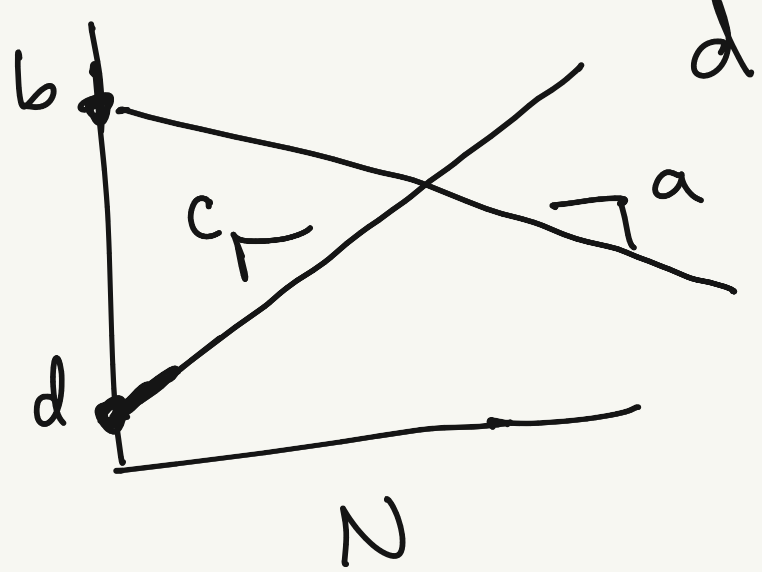
- <u>The logistic continuous-time population dynamic model</u> (Logistic)

Captures the idea that resources are finite and as

space food water nutrients

populations grow, ~~they~~ if runs out of resources to fuel its growth

- As ~~as~~ resources become more limiting, $\begin{cases} \text{birth rates decline} \\ \text{death rates increase} \end{cases}$



$d' = \underline{d} + cN$

$b' = b - aN$

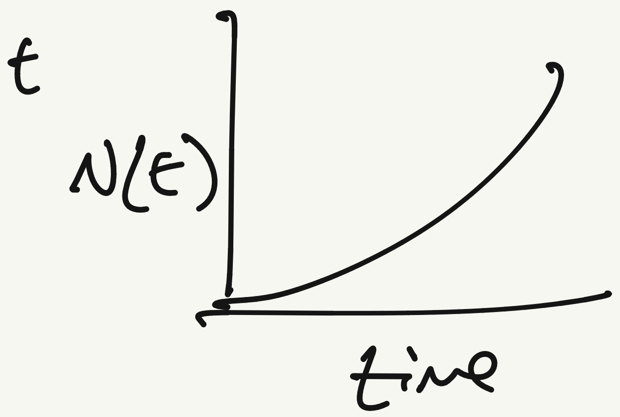
$\left\{ \begin{array}{l} c \\ a \end{array} \right.$ slopes ~ sensitivity of per-capita birth/death rates on population size N

$$\begin{cases} b' = b - aN \\ d' = d + cN \end{cases}$$

Exponential population growth

$$\frac{dN}{dt} = \underset{\overset{\uparrow}{(b-d)}}{r}N$$

sol: $N(t) = N_0 e^{rt}$



Substitute $b \to b'$

$d \to d'$

$$\frac{dN}{dt} = (b'-d')N = \left[\underbrace{(b-aN)}_{b'} - \underbrace{(d+cN)}_{d'}\right]N$$

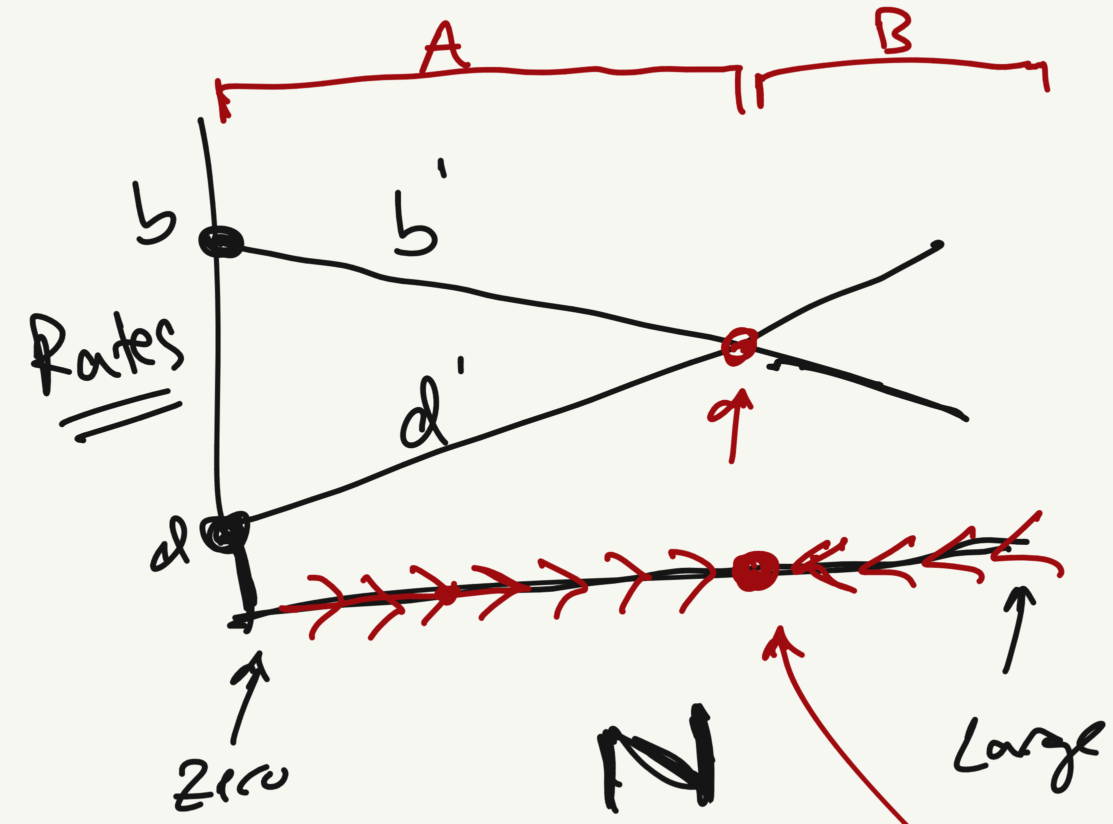
$$= \left[(b-d) - (a+c)N\right]N$$

$$\boxed{\frac{(b-d)}{(b-d)} = 1}$$

$$= \left(\frac{b-d}{(b-d)}\right)\left[(b-d) - (a+c)N\right]N$$

$$= (b-d)\left[\frac{(b-d)}{(b-d)} - \frac{(a+c)}{(b-d)}N\right]N = (b-d)\left[1 - \frac{(a+c)}{(b-d)}N\right]N$$

$$\frac{dN}{dt} = (b-d)\left[1 - \frac{(a+c)}{(b-d)}N\right]N$$



$$\overbrace{b' = d'}$$

$$b - aN^* = d + cN^*$$

$$b - d = aN^* + cN^*$$

$$(a+c)N^* = b - d$$

$$N^* = \frac{b-d}{a+c}$$

$$\frac{b-d}{a+c} = K$$

$$\frac{a+c}{b-d} = \frac{1}{K}$$

A: $b' > d'$

B: $d' > b'$

$$\underbrace{N^* = K}$$

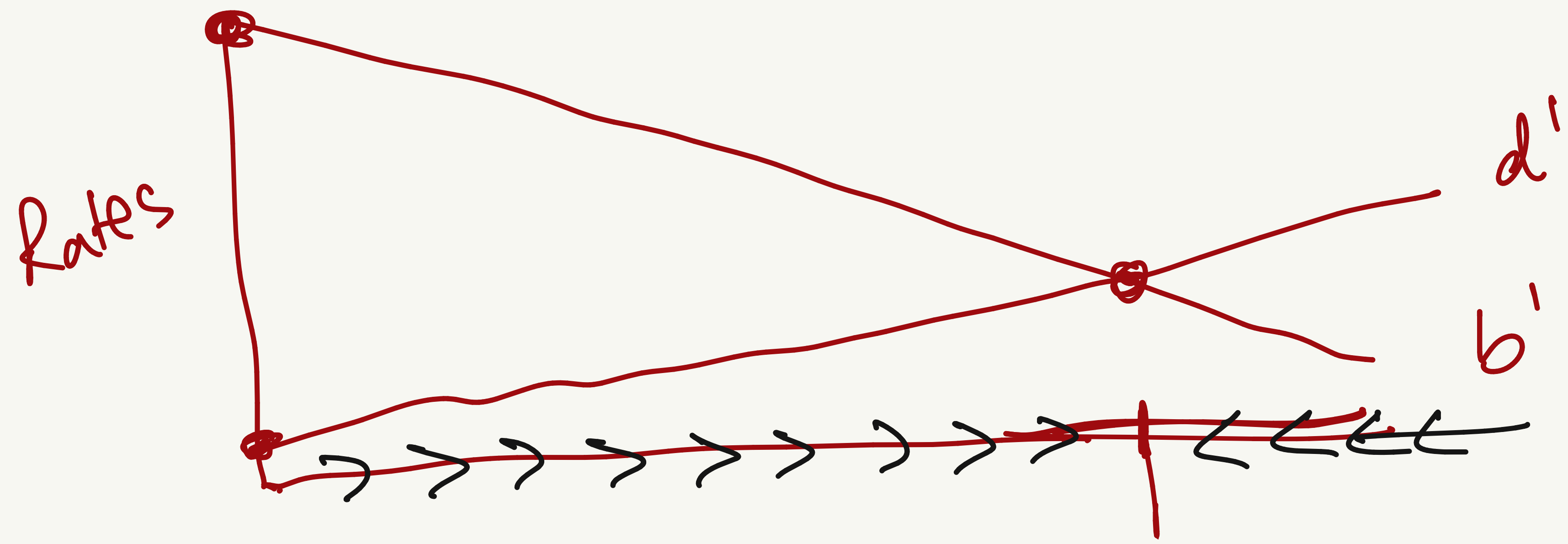
Stable steady state (attractor)

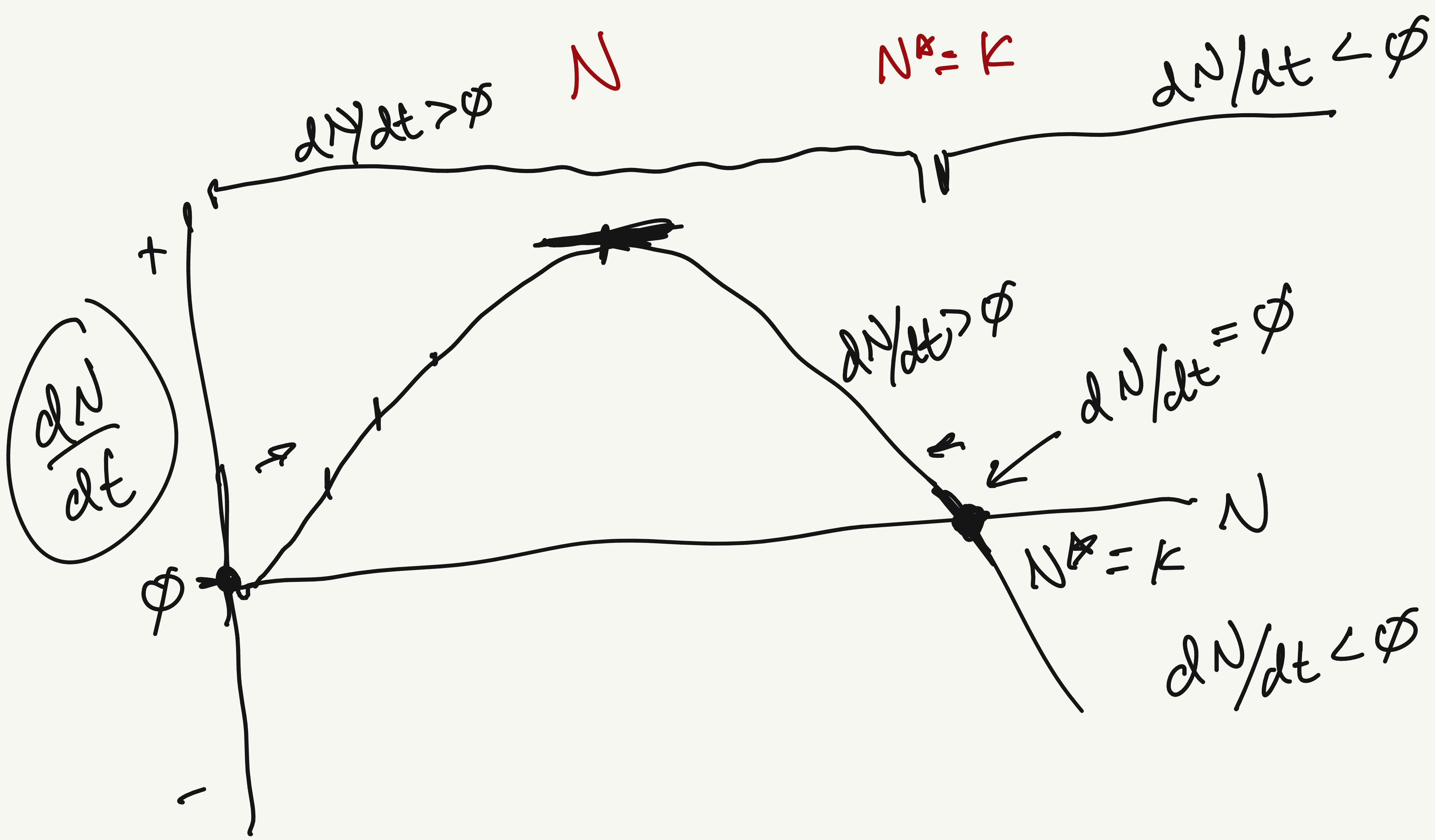
$N^* = K$

$$\frac{dN}{dt} = \overbrace{(b-d)}^{r}\left[1 - \frac{N}{\underline{K}}\right]N$$

Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \cdots\; rN - \frac{rN^2}{K}$$





$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

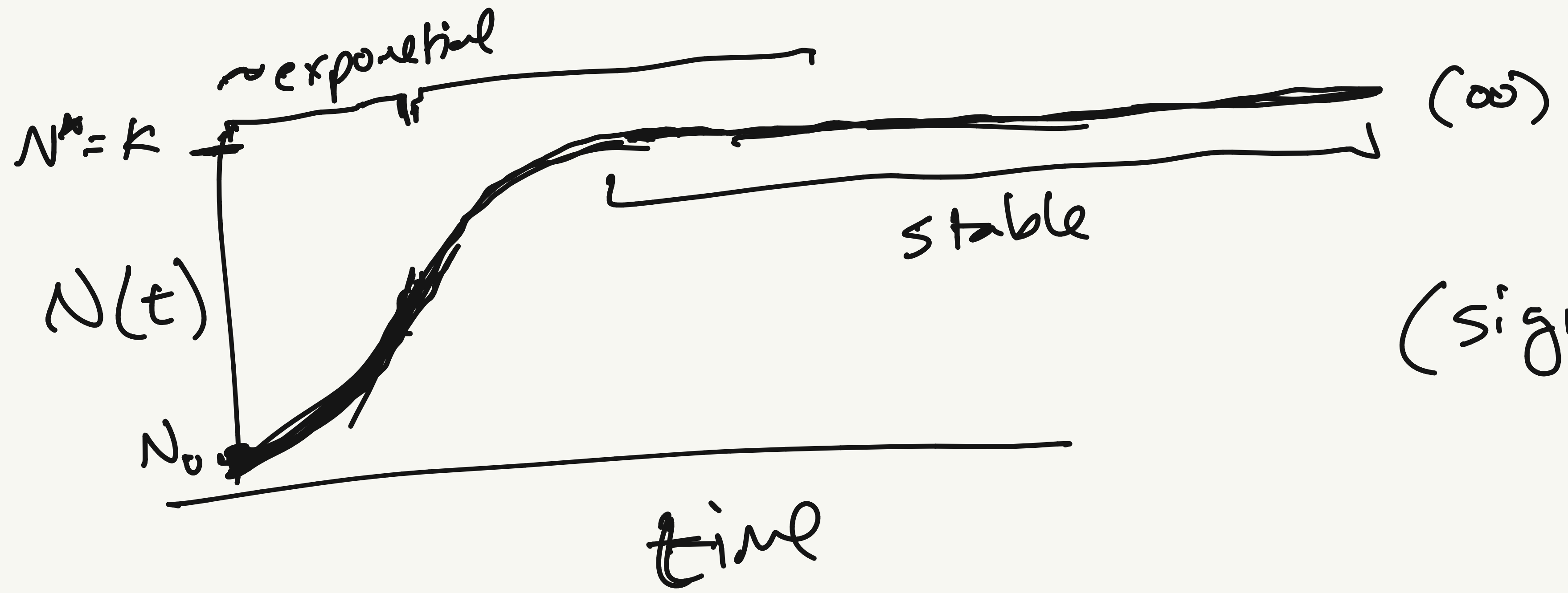
when $N \approx 0$

$$\frac{dN}{dt} = rN(1 - 0)$$

$$\frac{dN}{dt} \approx rN$$

$N \approx \emptyset$

$$\frac{dN}{dt} \approx rN$$



(sigmoidal curve)

$N \approx K$

$$\frac{dN}{dt} \approx rN\left(1 - \underbrace{\frac{\overset{(K)}{K}}{K}}_{1}\right)$$

$$\approx rN(1-1)$$

$$\frac{dN}{dt} \approx \emptyset$$

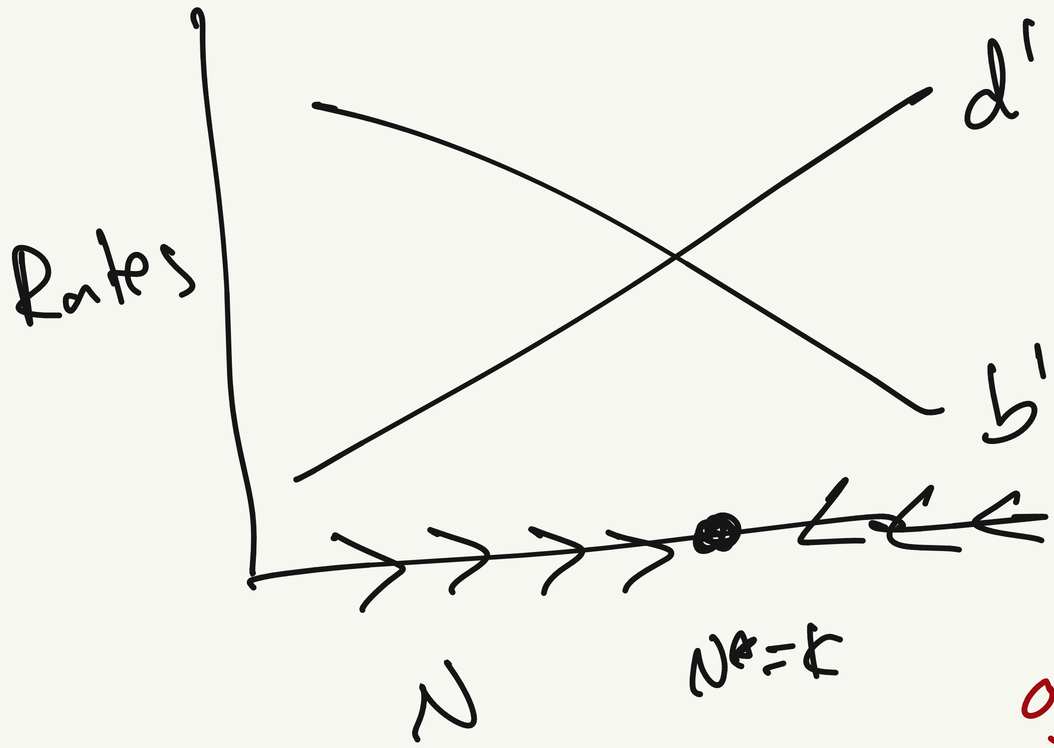
$$\frac{dN}{dt} = rN\left(1 - \boxed{\frac{N}{K}}\right)$$

serves as a "brake" on population growth as $N \uparrow$

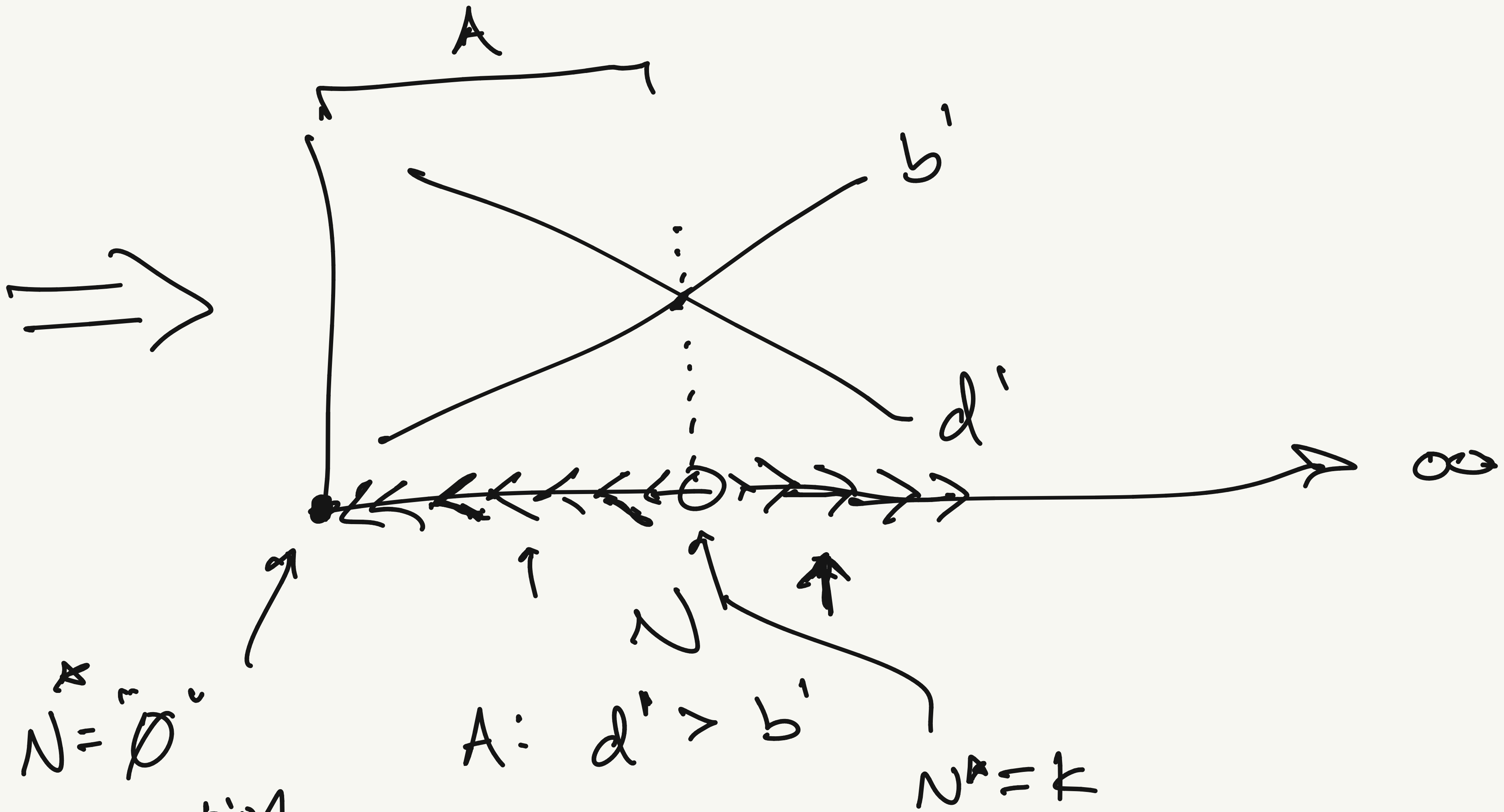
as $N \to K$

$$\frac{N}{K} \to 1$$

$$\left(1 - \frac{N}{K}\right) \to \emptyset$$



$\Longrightarrow$



A: d' > b'

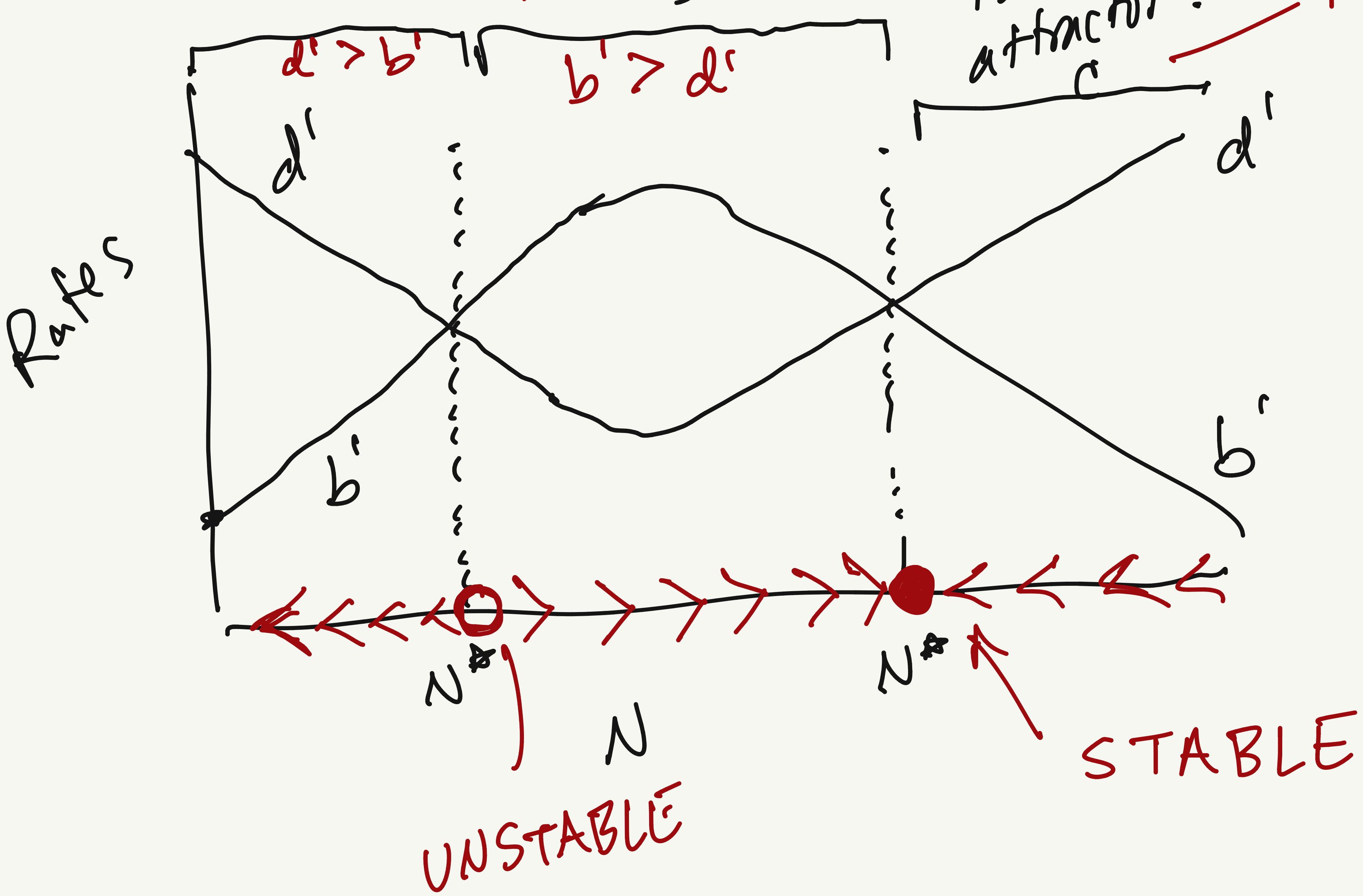
Extinction is an attractor!

Unstable ~~stab~~ steady state

(Pop. too small for group)

Benefits of group realized

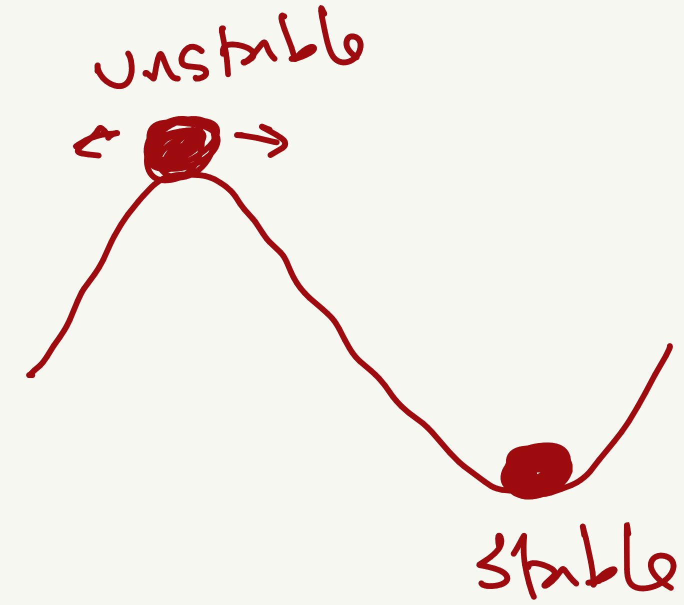
Resource limitation kicks in



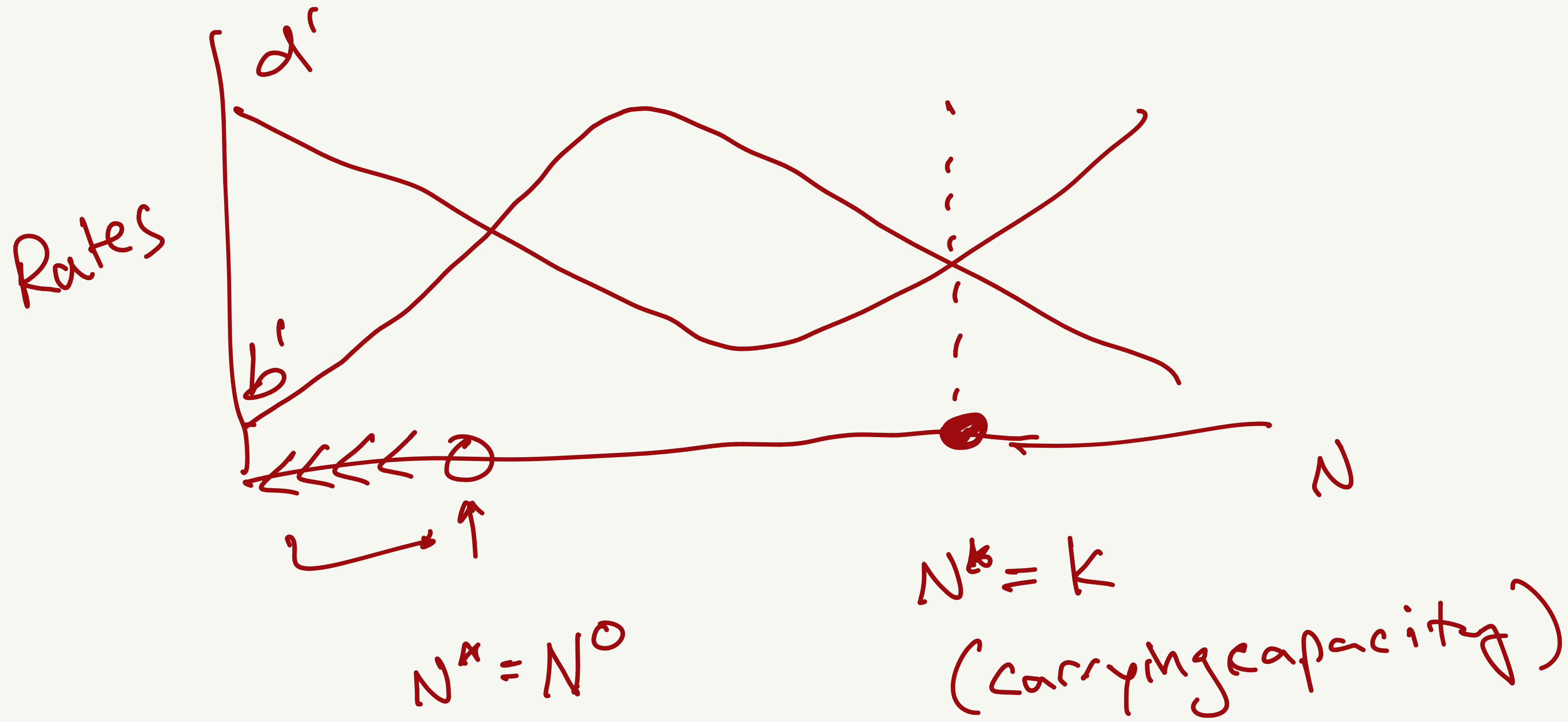
A: d' > b'

B: b' > d'

C: d' > b'



Consider benefits of groups



Allee Effect: when there is a critical minimum population size, below which extinction is inevitable