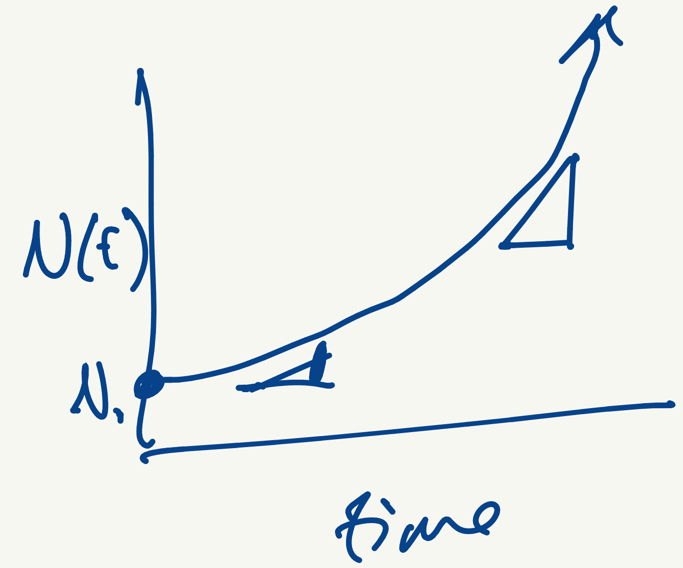
7 Billion humans

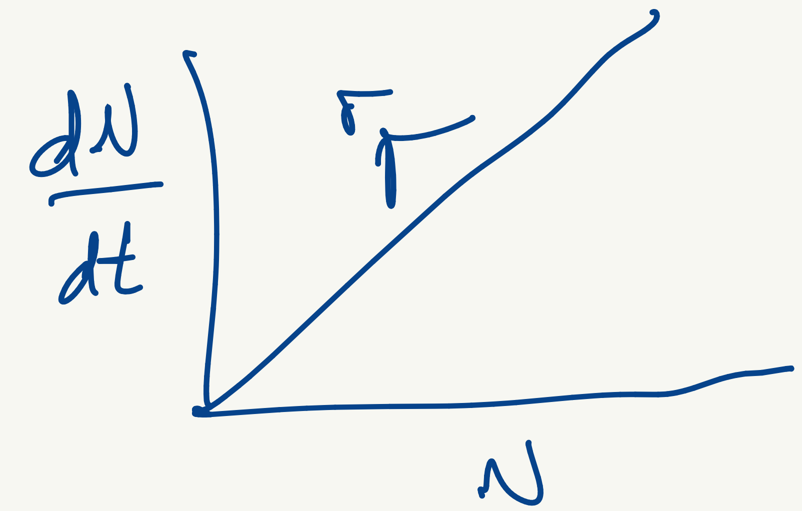
↓

3.5 Billion humans

$$\underbrace{\frac{dN}{dt} = \underline{rN}} \xrightarrow{\text{Calc.}} N(t) = \underline{N_0} e^{rt}$$



←



Doubling Time

$$\underline{2N_0} = \underline{N_0} e^{rt}$$

$$2 = e^{rt}$$

~~log~~ → <u>ln</u>

$$\log(2) = \log(e^{rt})$$

$$\log(2) = rt$$

$$t = \frac{\log(2)}{r}$$

(units)

$$r_{humans} = 0.0067 \; yr^{-1} \quad \frac{1}{[time]}$$

$$t = 103.4 \text{ yrs.}$$

Continuous vs. Discrete Population Growth

- seasonal mating cycles
  - ↳ Discrete more appropriate
- Imagine that population increases or decreases ~~eh~~ each year by a constant proportion:

$$N(t+1) = N(t) + r_d N(t) \qquad \overbrace{N(t+1) - N(t)}^{\Delta N} = \underbrace{r_d N(t)}$$

$$N(t+1) = N(t)\underbrace{(1 + r_d)}_{\lambda} \qquad \lambda = 1 + r_d$$

$$N(t+1) = N(t)\lambda \qquad \lambda = \frac{N(t+1)}{N(t)}$$

Ratio of population size at $t+1$ relative to pop. size at t

if $N(t+1) = N(t)$

$$\lambda = \frac{N(t)}{N(t)} = 1$$

if $N(t+1) > N(t)$

$$\lambda > 1$$

if $N(t+1) < N(t)$ $\quad \lambda < 1 \quad \lambda \geq \emptyset \qquad \emptyset \leq \lambda < 1$

Discrete

$N(t+1) = N(t)\lambda$

Recursion

$N(1) = N(\emptyset)\lambda$

$N(2) = N(1)\lambda$

$N(3) = N(2)\lambda$

$N(3) = [\underbrace{N(1)\lambda}_{N(2)}]\lambda$

$N(3) = \underbrace{[\underbrace{[\underbrace{N(\emptyset)\lambda}_{N(1)}]\lambda}_{N(2)}]\lambda}_{N(3)}$

$N(3) = N(\emptyset)\lambda^3$

$N(t) = N(\emptyset)\lambda^t$

Continuous

$\frac{dN}{dt} = rN$ $\quad N(t) = N_0 e^{rt}$

if $r = \emptyset$ $\quad \frac{dN}{dt} = \emptyset$

$(b = d)$

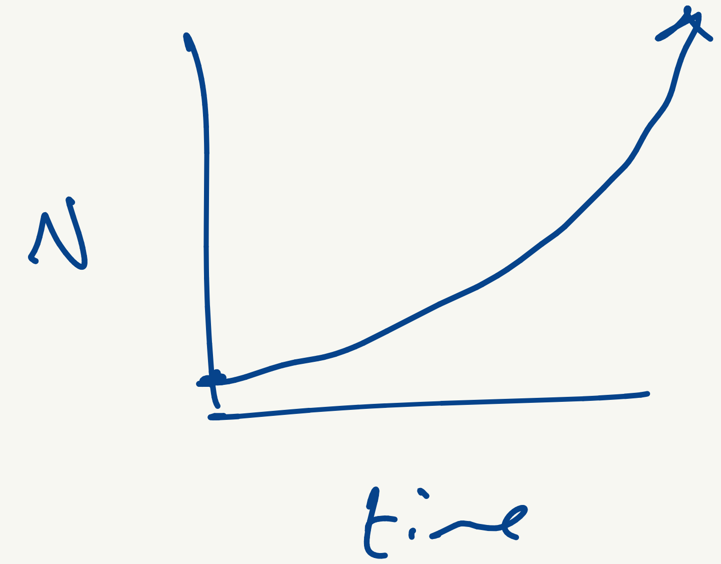
	Discrete	Continuous
Growth	$\lambda > 1$	$r > \emptyset$
Decline	$\emptyset \leq \lambda < 1$	$r < \emptyset$
Steady state	$\lambda = 1$	$r = \emptyset$

$$N(t) = \lambda^t N_0$$

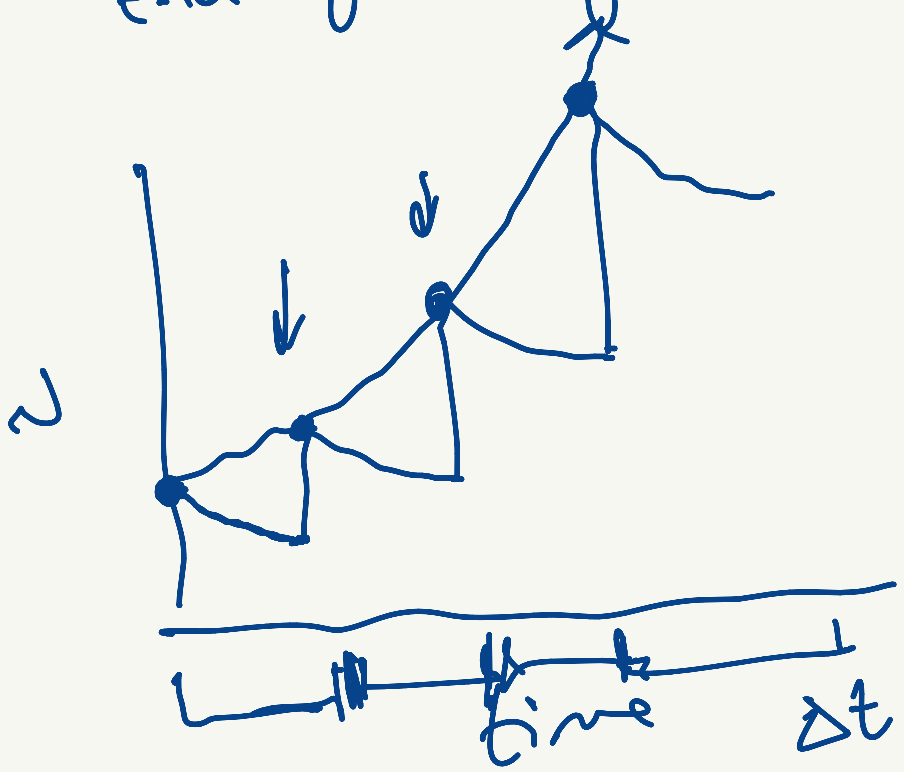
Initial population size

analogous

$$N(t) = N_0 e^{rt}$$



Births occur @ end of the year



Exponential Growth is unrealistic

$B = bN$

$D = dN$

Population Regulation

Density dependence

- Before, we assumed constant per-capita birth and death rates
- In reality, these per-capita rates are not constant and change by the size (density) of the population
- D.D. can be caused by:
  - Competition for resources (food, water, sunlight, space)
  - Prey-switching by predators
  - Disease w/ overcrowding

Continuous time

w/o D.D. birth/death rates

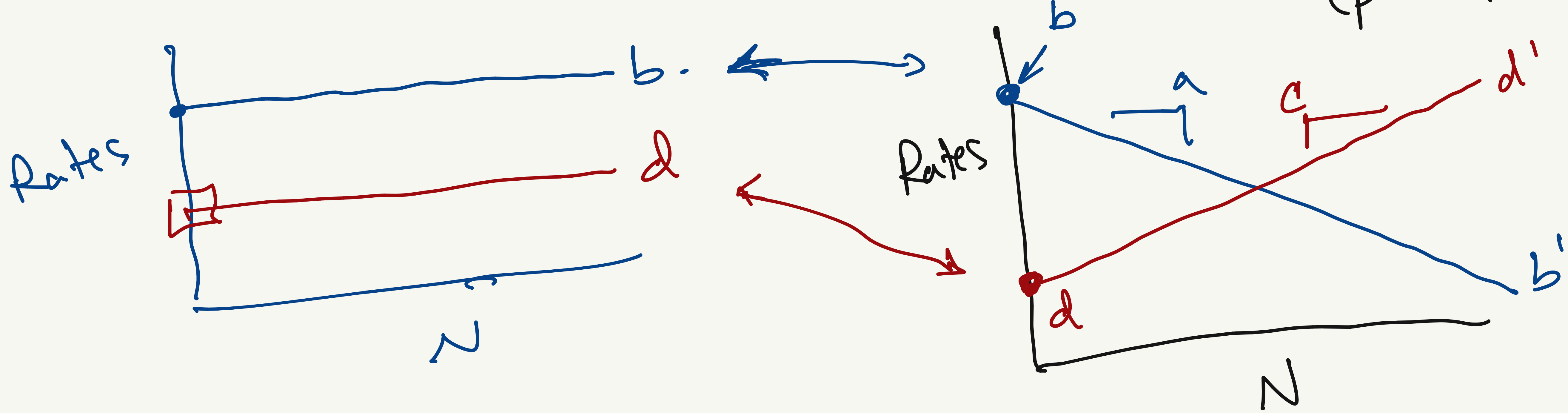
$$\frac{dN}{dt} = rN = (\bar{b} - \bar{d})N$$

$\bar{b} > \bar{d}$

Build in D.D.

Assume: births (per-capita) decline with N

deaths (per-capita) increase w/ N



(approx.)

b ~ per-capita birth rate when $N \approx \emptyset$

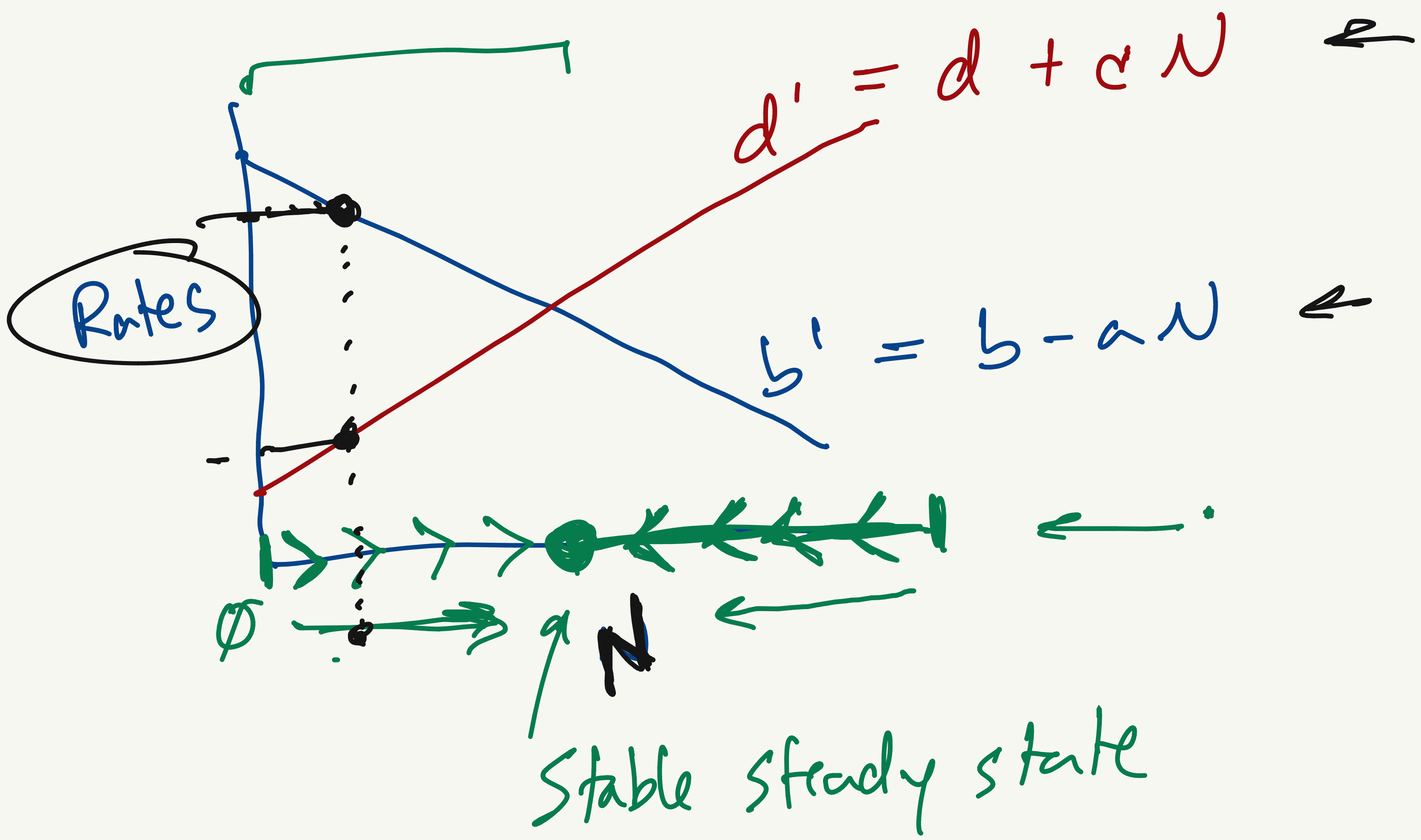
d ~ per-capita death rate when $N \approx \emptyset$

$$b' = b - aN$$

$$d' = d + cN$$

if slopes (a, c) are steep, small changes in N have big effects

on per-capita birth/death rates



- When birth rates > death rates
  N increases

- When birth rates < death rates
  N decreases

$$\frac{dN}{dt} = rN = (b' - d')N$$

$$\frac{dN}{dt} = \Big[(b - aN) - (d + cN)\Big]N$$

$$= \Big[\underbrace{(b-d)} - \underbrace{(a+c)N}\Big]N \quad \text{multiply by } \frac{b-d}{b-d}$$

$$= \left[\frac{(b-d)}{(b-d)}\right]\Big[(b-d) - (a+c)N\Big]N = (b-d)\left[\frac{(b-d)}{(b-d)} - \frac{(a+c)}{(b-d)}N\right] \times N$$

$$\frac{dN}{dt} = (b-d)\left[\underbrace{\frac{(b-d)}{(b-d)}} - \frac{(a+c)}{(b-d)}N\right]N$$

$$\frac{dN}{dt} = \underbrace{(b-d)}\left(1 - \underbrace{\frac{(a+c)}{(b-d)}}N\right)\underline{N}$$

$r \sim$ instantaneous population growth when $N \approx \emptyset$

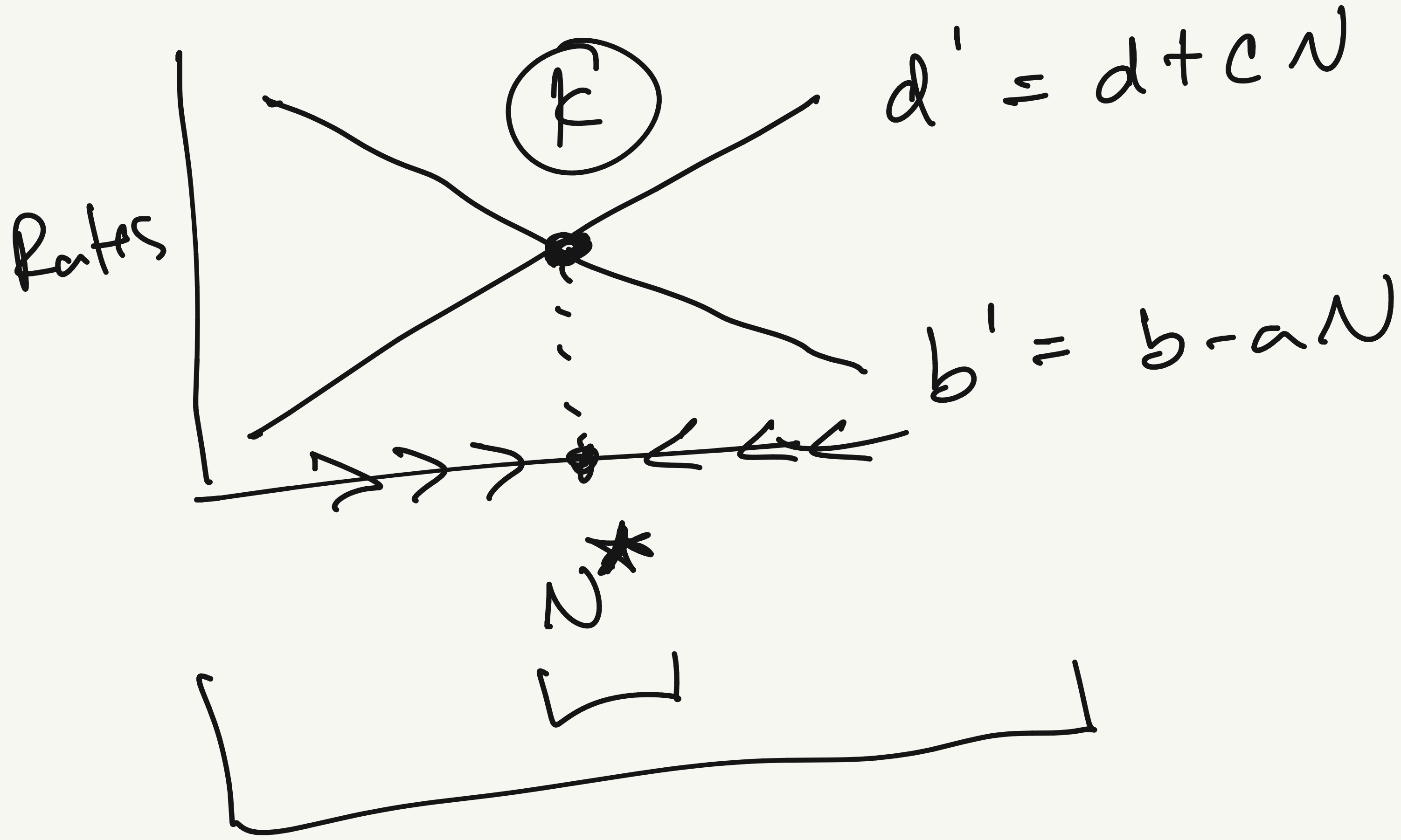
$$K = \frac{b-d}{a+c} \rightarrow \frac{1}{K} = \frac{(a+c)}{(b-d)}$$

when $N \approx \emptyset$ so $\frac{dN}{dt} \approx rN$

$$\frac{dN}{dt} = rN\left(1 - \underbrace{\frac{N}{K}}\right)$$

when $N \approx K$ so $\frac{dN}{dt} = \emptyset$

$$K = \frac{(b-d)}{(a+c)}$$ Carrying Capacity



$$d' = b'$$
$$d + cN^* = b - aN^*$$
$$cN^* + aN^* = b - d$$
$$N^*(c+a) = b - d$$
$$N^* = \frac{b-d}{a+c}$$

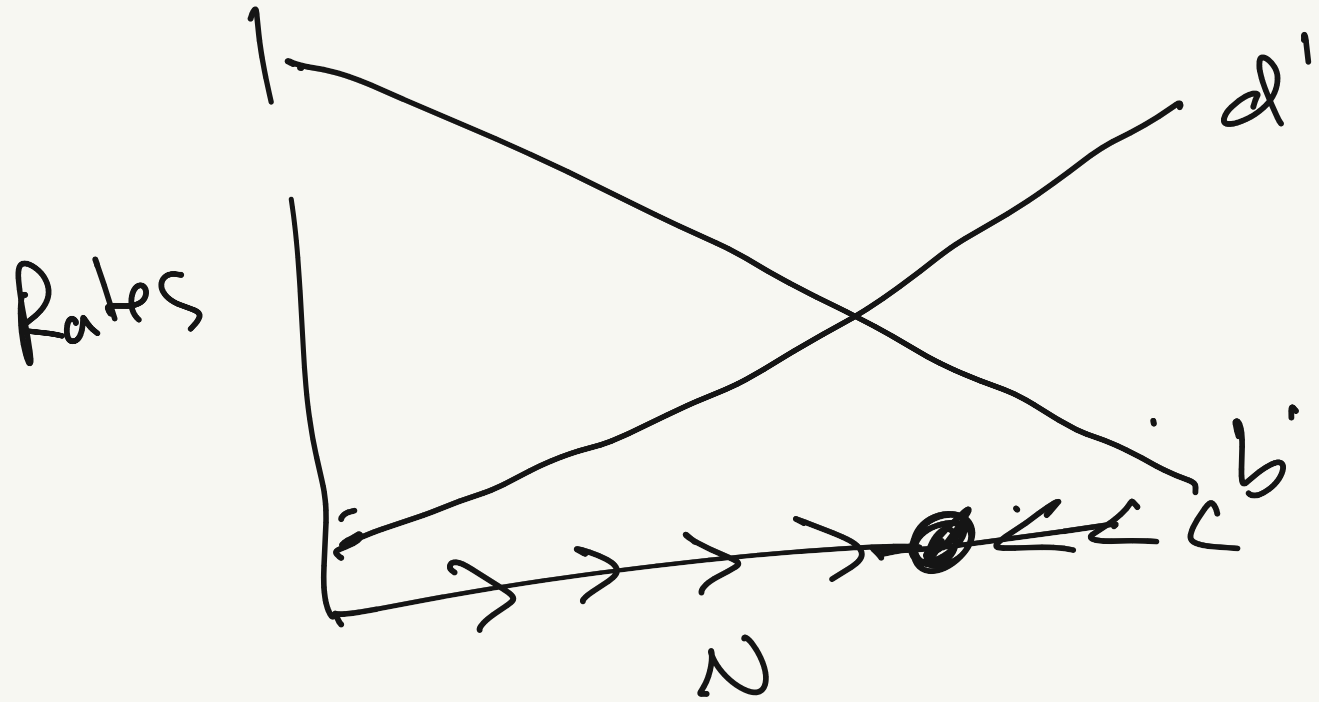
$$N^* = K$$

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

When $N = K$ $\quad \frac{dN}{dt} = \emptyset$

What does K mean?

- It is the $N(t)$ where births = deaths which means ~~more no~~ no more change



$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$