Fitness as result of interaction

Beetles < L ↑ energetic requirements
S ↓ energetic requirements

- when beetles of same size compete, they share what's left
- L Beetles overpower small Beetles

B2

focal Beetle B1	L	S
L	3	8
S	1	5

3/3	8/1
1/8	5/5

Beetles don't get to choose strategy... it genetically determined

Fitness differences drive changes in proportion of each phenotype in population $N_S(t)$ vs. $N_L(t)$

Evolutionary stable Strategy ~ Analagous to Nash Equilibrium

= genetically determined strategy that tends to persist once it is prevelant in population

Focal		A	B
	A	a	b
	B	c	d

	L	S
L	3	8
S	1	5

$$\Phi_A = a\frac{N_A}{N_T} + b\frac{N_B}{N_T}$$

$$\Phi_B = c\frac{N_A}{N_T} + d\frac{N_B}{N_T}$$

$$\Downarrow$$

$$\Phi_A = ax + b(1-x)$$

$$\Phi_B = cx + d(1-x)$$

$N_A \sim$ number of A inds. in population

$N_B \sim$ " " B "

$$N_T = N_A + N_B$$

$$\underset{x}{\frac{N_A}{N_T}} + \underset{(1-x)}{\frac{N_B}{N_T}} = 1$$

$$\Phi_L = 3x + 8(1-x) \rightarrow 8-5x$$

$$\Phi_S = 1x + 5(1-x) \rightarrow 5-4x$$

$\Phi_L = 8-5x$ y int: 8

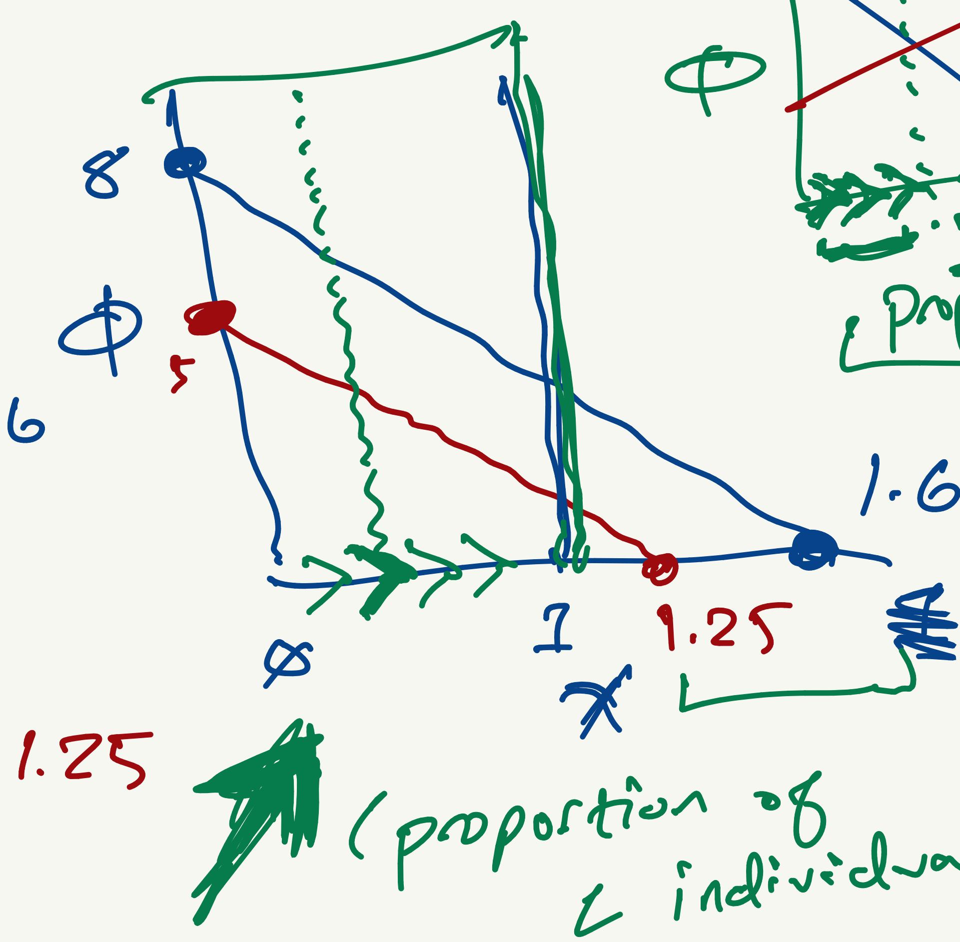
x int: $\phi = 8-5x \rightarrow 5x = 8$

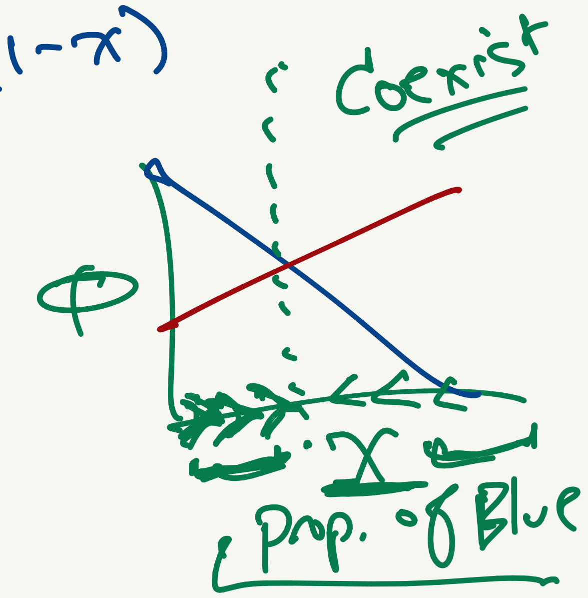
$x = 8/5 = 1.6$

$\Phi_S = 5-4x$ y int: 5

x int: $\phi = 5-4x \rightarrow 4x = 5$

$x = 5/4 = 1.25$





How do we relate fitness differences to changes in the population?

NOTE (A)

FYI: I changed the notation a bit compared to your section 6 (accidentally)

	Notes	Section
Morph fitness	ϕ	Φ
Average fitness	Φ	Ψ

proportion of L $= x$

" " S $= (1-x)$

Rule 1: If the fitness of a phenotype is better than average that proportion of that phenotype will increase

Rule 2: " " " " " " " worse " " " " " " " " " decline

Average fitness: $\Phi = x\phi_L + (1-x)\phi_S$

$$\rightarrow \quad \Delta x_L = x\left[\phi_L - \Phi\right] \longrightarrow \underbrace{x}\,\underbrace{(1-x)}\,\underbrace{(\phi_L - \phi_S)}$$

(+) if $\phi_L > \phi_S$

(-) if $\phi_L < \phi_S$

$$x(t+1) - x(t) = x\left[\phi_L - \Phi\right]$$

$$\underbrace{x(t+1)} = \underbrace{x(t)} + \underbrace{x\left[\phi_L - \Phi\right]}_{\Delta x}$$

~ Dynamic of the proportion of L phenotype in the population

$N(t)$ ~ Population size @ time t

$$N(t+1) = N(t) + \underset{\text{(births}}{B} - \underset{\text{(deaths)}}{D} + \cancel{I - E}$$

B ~ total number of births

D ~ total number of deaths

$B(N) = bN \qquad b = \dfrac{B}{N}$ per capita ~~#~~ birth rate

$D(N) = dN \qquad d = \dfrac{D}{N}$ per capita death rate

$$N(t+1) = N(t) + bN(t) - dN(t)$$

$$N(t+1) = N(t) + \underbrace{(b-d)}_{r_d} N(t)$$

r_d Avg. per-capita discrete growth rate

$$\overset{\Delta N}{N(t+1) - N(t)} = r_d N(t)$$

~ generalize to a time step of size Δt

Examine an interval Δt

$$N(t+\Delta t) - N(t) = \underbrace{r_d \Delta t}\, N(t)$$

Make our time window small

$$\frac{N(t+\Delta t) - N(t)}{\Delta t} = r_d N(t)$$

$r_d \sim$ discrete growth rate

Discrete time

↓

Continuous time

as $\Delta t \to \emptyset$

$$\frac{d}{dt} N = r N$$

"Change in population size over time"

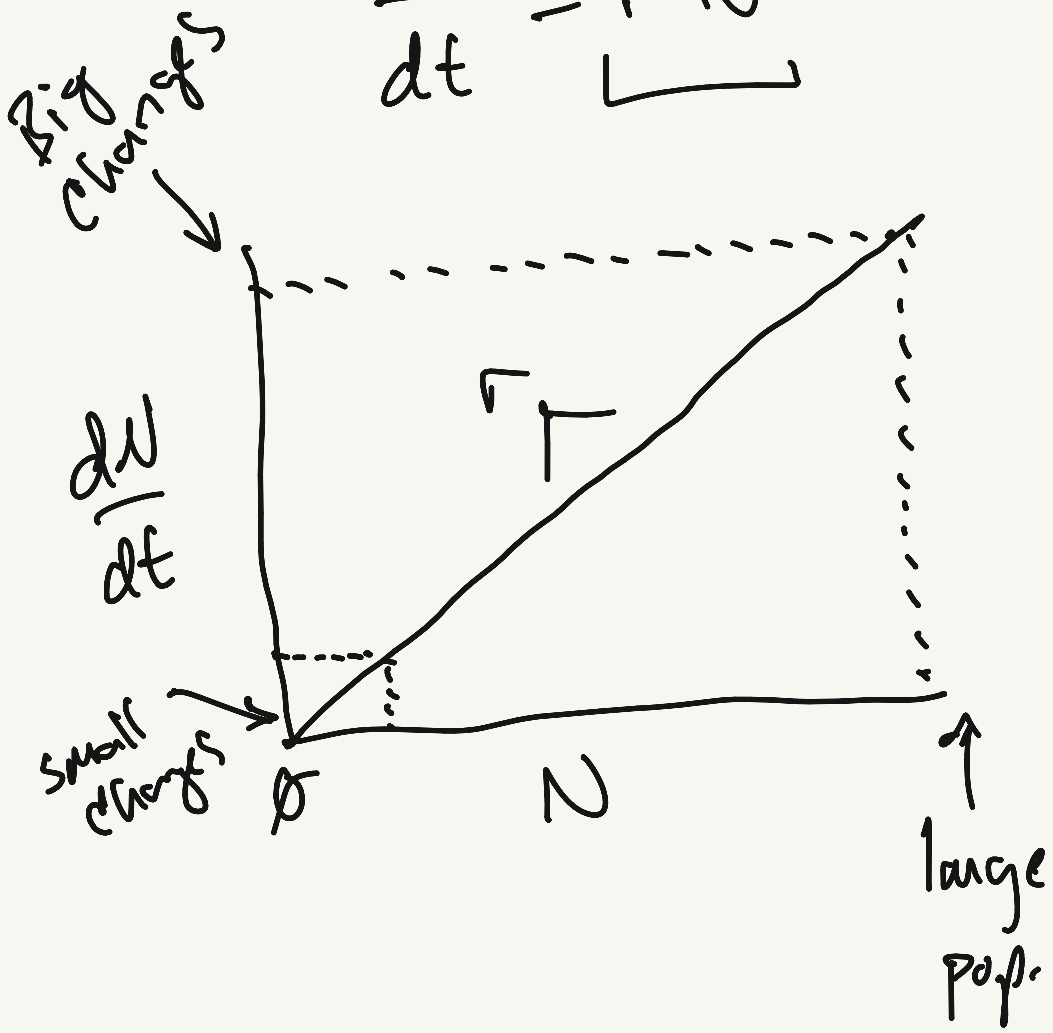
$r \sim$ instantaneous growth rate

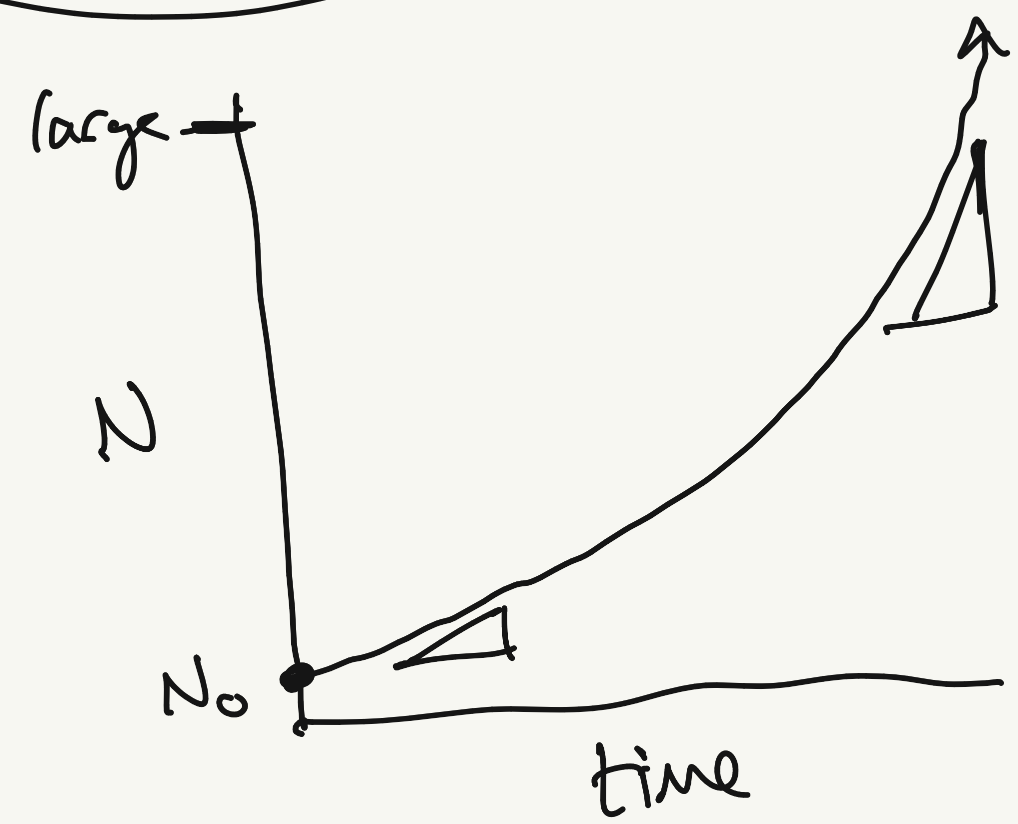
if $r > \emptyset$ $\quad \frac{dN}{dt} > \emptyset$ $\sim$ growth

if $r < \emptyset$ $\quad \frac{dN}{dt} < \emptyset$ $\sim$ decline

if $r = \emptyset$ $\quad \frac{dN}{dt} = \emptyset$ $\sim$ population not changing

$$\frac{dN}{dt} = \underbrace{rN}$$ SOLVE $$N(t) = N_0 e^{rt}$$





Ex) Doubling time

$$2\cancel{N_0} = \cancel{N_0} e^{rt}$$

$$2 = e^{rt}$$

$$\log(2) = rt$$

$$t = \frac{\log(2)}{r}$$

$$r_{Human} = 0.0067\ yr^{-1}$$