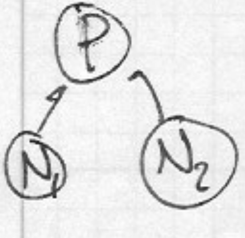
10.1



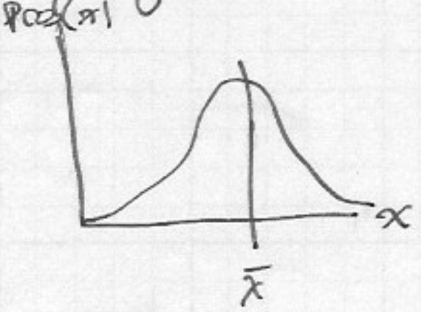
$$\frac{dP}{dt} = \sum_{i=1}^{2} e_i a_i N_i P - dP$$

$$\frac{dN_i}{dt} = r_i N_i\left(1 - \frac{N_i}{k}\right) - a_i N_i P$$

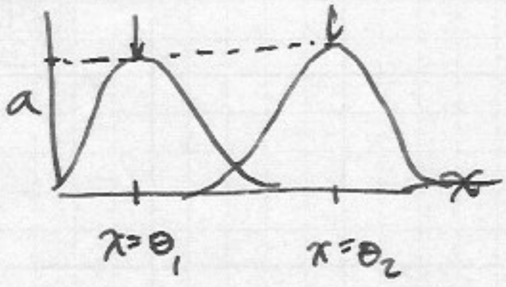
a_i ~ attack rate

- Assume that it depends on trait x
  - high → P is better @ attacking N_1
  - low → P is better @ attacking N_2

- $\underbrace{P(x,\bar{x})}_{\text{Probability that } x = X \text{ given mean } \bar{x}} = \text{Normal}(\bar{x}, \sigma)$



We can also assume that each prey has an optimal trait value at which attack rate is maximized



~ tradeoff... you can't be great @ both

- x is a distribution w/ mean & variance
  ~ many individuals w/ diff. x values (dist ~ population)

So: the mean attack rate of the predator on population i is:

$$\bar{a}_i(\bar{x}_i) = \underbrace{\int_{-\infty}^{\infty} a_i(x)\, p(x,\bar{x})\, dx}$$

sum over attack rates for trait value x given the probability that $x = \bar{X}$

$$\bar{a}_i(\bar{x}_i) = \int_{-\infty}^{\infty} \alpha_i \exp\left[\frac{(x-\theta_i)^2}{2\tau_i^2}\right] p(x,\bar{x})\, dx$$

$$= \frac{\alpha \tau_i}{\sqrt{\sigma^2+\tau_i^2}} \exp\left[\frac{(\bar{x}-\theta_i)^2}{2(\sigma^2+\tau_i^2)}\right]$$

Going back to eqns

Fitness: Per capita growth-mortality rate

$$W = \frac{1}{P}\frac{dP}{dt}$$

$$= \sum_{i=1}^{2} e_i a_i(x) N_i - d$$

and mean fitness is

$$\bar{W} = \int_{-\infty}^{\infty} W(x, N_1, N_2)\, p(x,\bar{x})\, dx$$

$$= \sum_{i=1}^{2} e_i \bar{a}_i(\bar{x}) N_i - d$$

$$\frac{dN_i}{dt} = r_i N_i\left(1-\frac{N_i}{K}\right) - P\bar{a}_i N_i$$

$$\frac{dP}{dt} = P\bar{W} = P\left(\sum_{i=1}^{2} e_i \bar{a}_i(\bar{x}) N_i - d\right)$$

$\frac{d\bar{x}}{dt} = \sigma^2_G \frac{d\bar{W}}{d\bar{x}}$ ← Change in mean trait value over time is proportional to

$$\left\{ \frac{\text{Change in mean fitness}}{\text{Change in mean trait value}} \right.$$

Stabilizing selection → Opposing force of selection

- $\bar{x}\uparrow$ but $\downarrow\bar{W}$, then $\frac{d\bar{x}}{dt} < 0$
- $\bar{x}\downarrow$ but $\bar{W}\downarrow$, then $\frac{d\bar{x}}{dt} > 0$

Directional selection

- $\bar{x}\uparrow$ and $\bar{w}\uparrow$, then $\frac{d\bar{x}}{dt} > 0$
- $\bar{x}\downarrow$ and $\bar{w}\uparrow$, then $\frac{d\bar{x}}{dt} < 0$

positive feedback!

Heritability

$h^2 = \frac{\sigma^2_G}{\sigma^2}$ } proportion of trait variation that is heritable

σ^2_G (genetic variance) fuels strength of selection.

if ↓variance, less to select 'for', and evol. slows down.

if ↑variance, more to select for, and evol. speeds up!