1.1 Ecological Dynamics

Planned topics to cover:

- Population dynamics
- Stability of systems (LSA)
- Bifurcations
- Generalizations into N-D systems (Matrix theory)
  - ~ Food webs
  - ~ Networks
  - ~ Perturbations
  - ~ Space
- Evolutionary dynamics
- Foraging ~ optimal decision making
- Fitness-based decision making
  - ~ stochastic environments
- Stochastic dynamics
- Ecodynamics symposium

Probability & stochasticity

2.1 Ecological Dynamics

Consider a population w/o age structure

$N(t)$ ~ population size in year t

$N(\emptyset)$ is known

λ ~ per capita growth rate $= \dfrac{N(t+1)}{N(t)}$

$N(t+1) = \lambda N(t)$ and we can show that $N(t) = \lambda^t N(\emptyset)$

$N(t+2) = \lambda N(t+1)$ $\quad \lambda < 1$ decline
$\quad\quad\quad = \lambda \lambda N(t)$ $\quad \lambda > 1$ growth

} a measure of fitness

Why?

w/ Density dependence

w/o Density dependence

Spatial Variation:

2 kinds of patches (habitats) < (poor) Patch 1 λ_1 fraction p ; (rich) Patch 2 λ_2 fraction $1-p$

# of inds in poor habitat : $pN(t)$
rich habitat : $(1-p)N(t)$

Next year:

$$N(t+1) = (\lambda_1 p N(t) + \lambda_2 (1-p) N(t)) = \underbrace{\{p\lambda_1 + (1-p)\lambda_2\}}_{\text{What is this?}} N(t)$$

An average

If we had n habitats, what is the average per-capita growth rate? p_i, λ_i

↳ $\sum_{i=1}^{n} p_i \lambda_i$ => Arithmetic Average

"If variation occurs over space, the arithmetic average is the appropriate description of the average growth rate"

Temporal Variation

- Assume per capita growth rate varies over time.
- With probability p_1 every individual experiences poor growth rate, $1-p$ rich growth rate.
- t is big... t_1 years for p ; t_2 years for $1-p$

~~Good~~ Bad $N(1) = \lambda_1 N_\phi$
Bad $N(2) = \lambda_1(\lambda_1 N_\phi)$
Good $N(3) = \lambda_2(\lambda_1 \lambda_1 N_\phi)$

$$N(t) = (\lambda_1)^{t_1} (\lambda_2)^{t_2} N(\phi) \quad \text{from } N(t) = \lambda_1\lambda_1\lambda_1\,\lambda_2\lambda_2\lambda_2 N(\phi)$$

if t is large, t_1 & t_2 ~ fraction of years that are bad, good.

i.e. $t_1 \sim pt$
$t_2 \sim (1-p)t$

so p is proportion of time spent in t_1

$$N(t) = \lambda_1^{pt} \lambda_2^{(1-p)t} N(\phi) = \underbrace{\left[\lambda_1^{p} \lambda_2^{1-p}\right]}^{t} N(\phi)$$

Different kind of average
~ Geometric mean
good & bad years are weighted differently than arithmetic average.
~ think of extreme case where $\lambda_1 = \phi$

Generalize over 'n' kinds of years:

$$N(t) = \prod_{i=1}^{n} \lambda_i^{p_i t} N(\phi)$$

Recall given $N(t) = \lambda^t N(\phi)$ and $\lambda = \exp(\log(\lambda))$

then $N(t) = e^{[\log(\lambda)]t} N(\phi)$

Define $r = \log(\lambda)$

then $N(t) = N(\phi) e^{rt}$

If time is continuous, this looks like population growth satisfying $dN/dt = rN$ w/ r as growth rate

- ~~Email to Jon, Rob, JP about reviews~~
-

Spatial Variation

$N(t+1) = N(t) \underbrace{\sum_{i=1}^{\hat{}} p_i \lambda_i}$ ← arithmetic average

combined per capita growth rate for spatially varying populations

Temporal Variation

$N(t) = N(\emptyset) \underbrace{\prod_{i=1}^{\hat{}} \lambda_i^{p_i t}}$ where ~~p_i~~ $t_i \sim pt$

good & bad years are weighted differently

Recall original: $N(t) = \lambda^t N(\emptyset)$

$\lambda = \exp(\log(\lambda))$

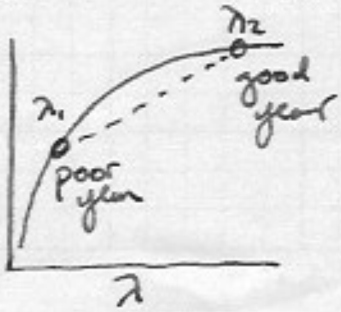
$N(t) = e^{[\log(\lambda)]t} N(\emptyset)$

Define $r = \log(\lambda)$

$N(t) = N(\emptyset) e^{rt}$ → looks like! $\frac{dN}{dt} = rN$, $N = N_0 e^{rt}$

⊛ what happens when $\lambda_1 = \emptyset$ in the spatial temporal case?

$N(t+1) = \underbrace{N(t)[p_1(\emptyset) + p_2(\lambda_2)]}_{> \emptyset}$ vs. $N(t) = \underbrace{[(\emptyset)^p \lambda_2^{1-p}]^t N(\emptyset)}_{\emptyset}$



growth rate as a function of λ

- The growth rate at the arithmetic average of λ is <u>larger</u> than the average value of ~~the~~ 2 growth rates.

2.4a

Rewrite $N(t) = N(\emptyset) \prod_{i=1}^{\hat{}} \lambda_i^{p_i t}$ in terms of logarithms of the per-capita growth rates

$$N(t) = \exp\left[t \sum_{i=1}^{\hat{}} p_i \cdot \log(\lambda_i)\right] N(\emptyset)$$

from $N(\emptyset)\left[\lambda_1^{p_1 t} \lambda_1^{p_2 t}\right]$

$$= N(0) \exp\left[\log\left[\lambda_1^{p_1 t} \lambda_2^{p_2 t}\right]\right]$$

$$= N(\emptyset) \exp\left[p_1 t \log(\lambda_1) + p_2 t \log(\lambda_2)\right]$$

generalize $= N(\emptyset)\left[\exp\left[t \sum_{i=1}^{\hat{}} p_i \log(\lambda_i)\right]\right]$

given $N(t) = N(\emptyset) e^{rt}$

then $r_{temp} = \sum_{i=1}^{\hat{}} p_i \log(\lambda_i)$

r_{temp}

~ for temporal variation (i.e. a fluctuating environment), the growth rate is the arithmetic avg of the logarithms of the per-capita growth rates

And the spatial form

$$N(t+1) = N(t) \underbrace{\sum_{i=1}^{\hat{}} p_i \lambda_i}_{\text{Avge } \lambda}$$

Avge λ -> convert to r

$r = \log(\lambda)$

$$N(t+1) = N(t) \exp\left\{\log\left(\sum_{i=1}^{\hat{}} p_i \lambda_i\right)\right\}$$

$r_{spat.}$

$$r_{spat} = \log\left(\sum_{i=1}^{\hat{}} p_i \cdot \lambda_i\right)$$

so we have

spatial growth rate: $r_{spatial} = \log\left(\sum_{i=1}^{n} p_i \lambda_i\right)$

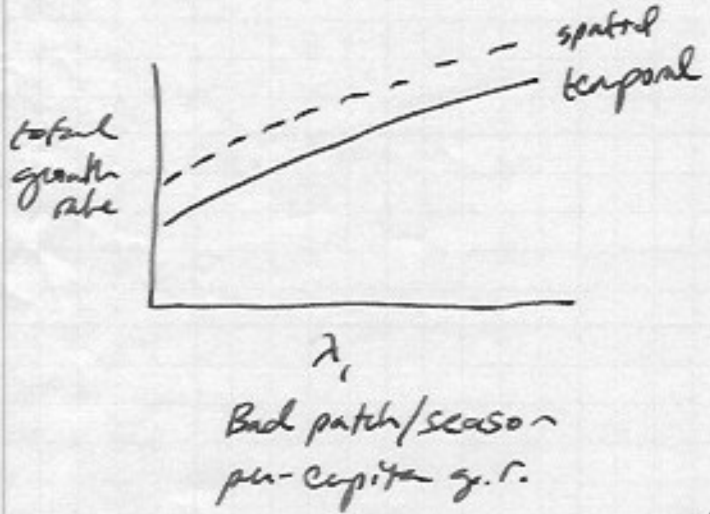
logarithm of the arithmetic average of patch-specific per-capita growth rates.

temporal growth rate: $r_{temporal} = \sum_{i=1}^{n} p_i \log(\lambda_i)$

arithmetic average of the logarithm of time-specific per-capita growth rates

What does this mean??

2 patch / 2 season growth rate comparison:



$\lambda_2 = 2 \cdot \lambda_1$

$p_1 = ~~0.5~~, 0.2$

$p_2 = (1 - p_1)$

(higher fitness)

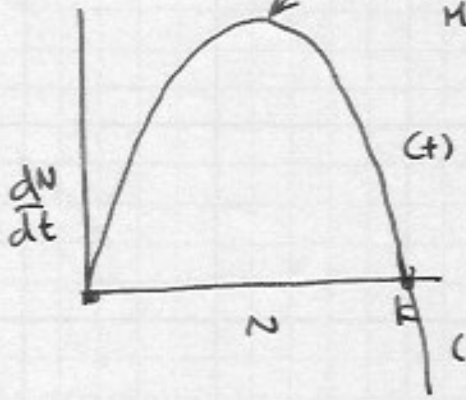
* Variation in space → higher r compared to similar variation over time

* Deep concepts from the simplest population model

$N(t+1) = \lambda N(t)$

The Logistic Equation:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

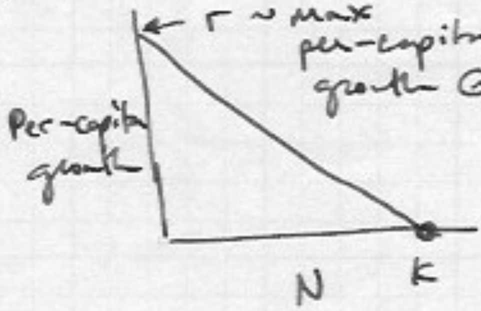


Max net productivity
MNP @ $N = \frac{K}{2}$

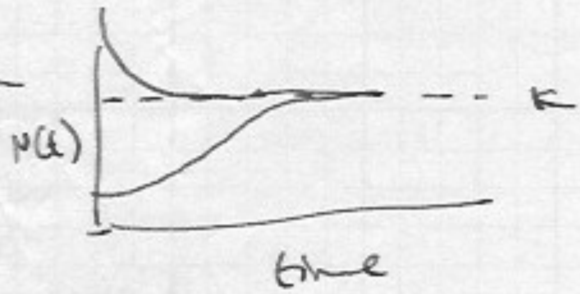
by $\dfrac{d\left(rN\left(1-\frac{N}{K}\right)\right)}{dN} = \emptyset$

per-capita growth:

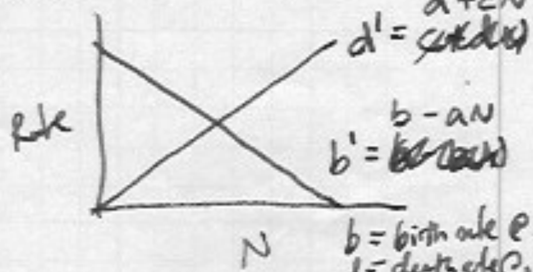
$$\frac{1}{N}\frac{dN}{dt} = r\left(1 - \frac{N}{K}\right)$$



← r ~ max per-capita growth @ $N = \emptyset$ (No inhibitions but not realistic)



Take-home problem



b = birth rate @ $N \to \emptyset$
d = death rate @ $N \to \emptyset$

$$\frac{dN}{dt} = \left[b'(N) - d'(N)\right]N$$

given $r = b - d$

derive $\dfrac{dN}{dt} = rN\left[1 - \dfrac{N}{K}\right]$

& what is K?

From continuous to discrete!

$$\lim_{dt \to \emptyset} \frac{N(t+dt) - N(t)}{dt} = rN\left(1 - \frac{N}{K}\right)$$

$$\Rightarrow N(t+1) = N(t) + rN(t)\underbrace{\left[1 - \frac{N(t)}{K}\right]}$$

Problem: if $N(t) >> K$ .... is negative,

if r is very large, $N(t+1)$ might be $< \emptyset$!

Alt. $N(t+1) = N(t)\exp\left\{r\left(1 - \frac{N(t)}{K}\right)\right\}$

Explore Cobweb Diagram!

Intuition: if $\boxed{\text{max}}$ per-capita reproduction is A

s.t. $N(t+1) = AN(t)$

- a focal offspring has prob. 'f' ~~f~~ of surviving when one ~~offsp~~ adult is present
- N adults present
- Prob (survival) = f^N

SET
$$\boxed{f^N = \exp^{-bN}}$$

then: $N(t+1) = AN(t) f^{N(t)}$

set $f^N = e^{-bN}$

$N(t+1) = AN(t)e^{-bN(t)}$

$N\log(f) = -bN$

$\log(f) = -b$

$-\log(f) = b$

$$\underbrace{N(t)}_{\text{surviving offspring}} e^{-bN(t)}$$

~~$\log f = \Delta x$~~

~~A~~ so $A = \lambda$

$$AN(t)\,e^{-bN(t)} \;\; \| \;\; N(t)\exp\left\{r\left(1-\frac{N(t)}{K}\right)\right\}$$

=

$$A\exp\{-bN(t)\} = \exp\left\{r\left(1-\frac{N(t)}{K}\right)\right\}$$

(log) $\log(A) + \cancel{\log} -bN(t) = r\left(1-\frac{N(t)}{K}\right)$

$= r - \frac{rN(t)}{K}$

$\log(A) \sim r$

$b \sim \frac{r}{K}$