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APPLIED MATHEMATICIANS AND NAVAL OPERATORS*

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Dedicated to Philip Morse on the fortieth anniversary of the
Anti Submarine Warfare Operations Research Group

Abstract. This is a personal view of applied mathematics in a truly nonacademic setting (an operational navy command), based on the author's experiences as a field representative of the Operations Evaluation Group. The principles that can make analysis effective in such a setting are described and explained. The intermeshing of people and personality is as important as the fields themselves. The traits that a mathematician needs to effect change by his analysis are discussed. A number of specific examples are provided to illustrate the kinds of problems that can be encountered.

> Those who have learned only how to apply some given theoretical framework to the solving of problems which arise within this framework, and which are soluble within it, cannot expect that their training will help them much in another specialism. It is different with those who have themselves wrestled with problems, especially if their understanding, clarification, and formulation, proved difficult . . . those who have wrestled with a problem may be compensated by gaining an understanding of fields far removed from their own . . . there are no subject matters but only problems . . . which almost always need for their solution the help of widely different theories.
>
> —Karl R. Popper, *Objective Knowledge*
> (Oxford University Press), p. 182.

1. Introduction. Although this purports to be a paper about nonacademic applied mathematics, it is really about the interaction of applied mathematicians and naval operators. By "operator," I mean an individual whose job involves some part of naval action, e.g., navigating a ship, flying an aircraft, running a radar, etc. The wording is important, because I believe that when applied mathematics is done in a nonacademic setting, the intermeshing of people is as important as the fields or techniques involved. The nonacademic setting for me was an operational naval command at Whidbey Island, Washington, where I worked as the Operations Evaluation Group field representative for a period of 18 months. Hence this is a personal view, but I think that much of what will be said here translates to other fields with little or no change.

Very often in research, the actual process of doing research is more satisfying than the particular problem being studied. It is for this reason that nonacademic applied mathematics can be so rewarding. But there is a serious pitfall, one that should be acknowledged from the outset. It is difficult to describe this kind of work without its appearing to be trivial. And often the more applied a problem is, the more *mathematically* trivial it may be. This runs against the training we receive in mathematics that "harder is better."

On the other hand, in many applied problems the real difficulty and intellectual challenge is in finding and characterizing the problem in the first place, and not in solving it. The harder-is-better syndrome is a pitfall that can be overcome, once one is aware of it. Of course, one will not become "famous" in the applied mathematics community by solving problems that appear to be easy. This is another trade-off that must be considered.

In the next section, I describe the Operations Evaluation Group (OEG), one of the oldest organizations in the United States doing operations research and one of the few that is still problem, rather than technique, oriented. I discuss the principles that make

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OEG effective. In §3, I discuss the problems that naval operators face. They are generally operational not technical ones, and require useful, immediate solutions. In §4, I describe what I believe an applied mathematician can offer to the operators, in terms of approach and outlook. In §5, three examples of problems that I worked on are given. There are many other examples alluded to, but these three capture the essence of the work.

At the outset, I wish to thank the many officers and enlisted men and women who helped me, especially LCDR Bill Headridge and CAPT Paul Hollandsworth, whose efforts made my tour a success, and who are now good friends. I also thank RADM Henry Arnold, who as the commander at Whidbey created such a good working environment.

2. The Operations Evaluation Group. Operations research in the United States began in 1942, with the formation of the Anti Submarine Warfare Operations Research Group (ASWORG), headed by Philip Morse. The history of ASWORG is described in Morse's book [1]. Morse collected a group of scientists and mathematicians, including George Kimball, Bernard Koopman and William Shockley, to work on various problems associated with German submarine operations off the Atlantic Coast. The group was very successful; some of its work is described in reports issued in the late 1940's [2]–[4]. After World War II, the ASWORG became the Operations Evaluation Group (OEG), and in 1962 OEG and the Institute for Naval Studies were merged to form the Center for Naval Analyses (CNA).

Operations research was created in the turmoil of a war to solve problems. The point of ASWORG was to bring a scientific approach to naval problems, and not to find problems that could be solved by a certain set of techniques.

The breadth of the staff of ASWORG caused many different techniques to be used in the solving of problems of interest to the Navy. Today, operations research is codified, much to its detriment [5], [6]. When operations research is mentioned, people usually think of a set of techniques (e.g., linear programming, optimization, queuing theory, etc.) rather than a viewpoint about problem solving. This may be due, in part, to the introduction of academic departments of operations research [5], [6]. However, the old style, problem solving approaches of operations research are still being used by OEG, and other like organizations.

During the first few years of ASWORG, a number of principles for effective analysis emerged. Although these principles were never stated as such, they have been used to guide OEG for 40 years. My view of the principles is this:

1. *Principle of closeness: The analyst must be near the problem.* It was discovered early in ASWORG's work that the analysts could do much more for the Navy if they were at the scene of the problem—where they could see the equipment in operation and talk with the operators about the problems—than if they were at a desk 2000 miles from the problem. This observation lead to the OEG field program, in which analysts leave the home office for 1–3 years and go to work at the naval command. They are attached to the staff at the command and work as theoreticians for the operators.

2. *Building confidence.* In general, operators feel that analysts don't have much to offer them, and it is up to the analyst to find simple ways of convincing the operator that analysis is useful and that he (the analyst) cares about the problems of the operator. There are many different ways to build confidence. In my case, for example, I won the confidence of one operator by writing a program for the HP-69 pocket calculator that allowed him to do calculations on the machine in 30 minutes that had taken him 4 hours the day before; of another operator by showing how to use the central limit theorem to estimate how many trials of a test were needed to gain a certain accuracy; and of an entire

squadron of aviators by showing them how to calculate when a typhoon would hit the carrier we were on. I did this by fitting a simple quadratic to give distance between us and the typhoon as a function of time.

3. *Interaction at all levels.* The third principle established during World War II was that the analyst should be able to interact easily with all levels of the Navy. For this reason it is helpful if the analyst is a civilian, instead of an officer. Admirals make decisions, but it is the junior officers who know the operations of the equipment intimately, and it is the enlisted technicians who have insights into technical problems and into problems of maintenance and reliability. In my case, for example, I found it very important to consult with the enlisted men while designing data collection sheets for the analysis of the effectiveness of a new type of sensor. They guided me into areas that few of the officers were aware of.

4. *Hemibel thinking.* A complicated analysis that leads to a 2% increase in effectiveness of operations is generally not worth doing. In the early years of ASWORG, it was agreed that if the actual value of an operation was within a factor of 3 (i.e., 1 hemibel) of the theoretical value, then a change in operation would be unlikely to improve the result. Perhaps this overstates the case for present day operations analysis, but not by much. The goal of the analysis, after all, is understanding, prediction, and improvement in system performance.

3. Problems of naval operators. The problems faced by the operators can be characterized as follows: First, the problems are operational, not technical or design. The operator has a piece of equipment, which undoubtedly could be improved, but he needs to figure out how to use it today. An analyst who tells him how to improve the equipment without telling him how to use today's equipment only angers and alienates the operator. For example, the EA-6B is a sophisticated electronic warfare aircraft first built for use in Viet Nam. There are presently two versions of the aircraft used in the fleet; they differ mainly in the automated nature of detection and identification processes. Two other versions, which will be even more automated, are being designed. These new aircraft will reduce the operators' workload considerably. The operators, however, need tactics for the present day aircraft, not for the superior ones down the line.

The second characteristic of the problems is that although operators are looking for good ways of doing their jobs, optimizing some mathematical criterion is not their goal. This is especially important since most soluble mathematical optimization problems have little to do with the real operation. In H. Simon's language [7], [8], they are seeking to satisfice rather than optimize. An example of this phenomenon arises in search theory. Searches are conducted almost every day by naval operators, and starting in World War II, the theory of search developed into a mathematical subject [4], [9] of complexity and sophistication. In the development of search theory, the idea of optimal search has played a central role. Yet the operators often don't care if they have an *optimal* search plan; they just want one that is "not too bad." They need a figure like Fig. 1, and want to know if life follows curve A or curve B. This will determine how close to the optimal plan their plan needs to be, to be not too bad.

The third characteristic of the problems is that there are always stochastic factors present in the world, and often deterministic biases that may be larger, generally unknown and very hard to deal with. The presence of stochastic fluctuations and unknown deterministic biases makes it impossible to approach problems as if life were predictable and controllable from the outset. I shall provide two examples of this phenomenon in §5.

Two characteristics concern the operators themselves. They are not mathematicians, but after they are convinced that the analyst cares, they can often see the value of a well

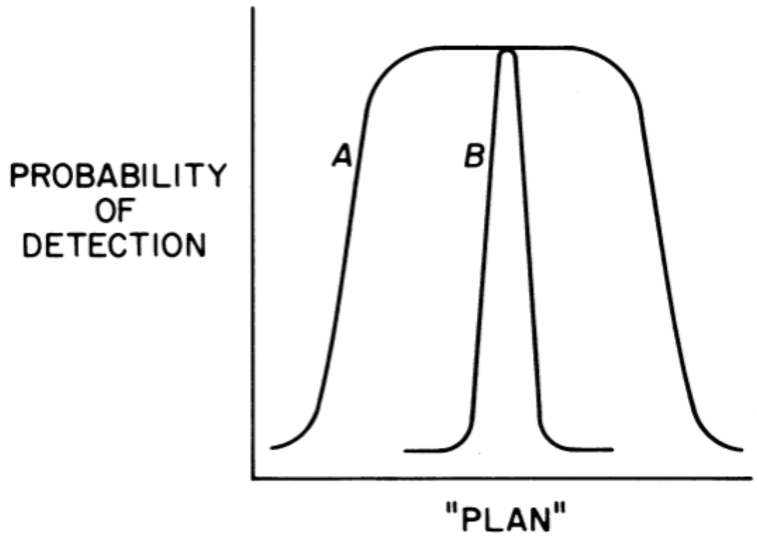

FIG. 1. *A schematic that the operators need, characterizing probability of detection and search plan.*

presented analysis. Often the problems tackled by the analyst require only algebra, elementary statistics and calculus. There are many officers who have these tools and lack the confidence to use them, but they can appreciate the analysis.

Finally, the good operators often have some sort of model of the problem of interest. It is usually a verbal model and it is up to the mathematician to draw out the model and convert it to a mathematical one. For example, in air to air combat, pilots had for many years a system for classifying the state of the combat according to the relative positions of the aircraft (highly advantageous for aircraft 1, advantageous, neutral, etc.). The pilots also had ideas about transitions between states. My colleagues R. Oberle, W. Nunn and S. Naron converted the verbal model to one modeled using a semi-Markov process [10]. Their analytical methods are now in use on both coasts for the training of aircrews in air combat maneuvering.

4. What can an applied mathematician offer? By its very nature, applied mathematics is difficult to define, and since this is supposed to be a paper about people, I will instead consider what an applied mathematician can offer. His first ability is that of model building. This means taking the verbal model of the operator and converting it to a mathematical one. The crucial step here is knowing what terms to throw away and what to keep when creating the model. Once a model is constructed, qualitative analysis, approximations, and interpretations of the analysis are all trademarks of the applied mathematician. Another trait is the ability to obtain numbers. A facility for computing is important, since the operators want answers, and these are usually numerical in nature. Some knowledge of statistics is helpful.

Throughout any work in this type of setting, the goal of the applied mathematician is change. One is successful if he or she can change the way a problem is viewed or the way an operation is done. Too often, in too many settings, the goal of change is overlooked [11].

The final, and perhaps most important, characteristic that the mathematician needs is the desire to solve problems. One cannot be successful in this kind of work by bringing techniques and looking for problems that can be solved by those techniques. The problems are paramount, and one needs the versatility to bring in any techniques that can be used to solve them.

The particular academic training, in courses and in research, is not as important as the characteristics described above. One has to be willing to learn more and to learn whatever is needed for the job.

5. Some examples of problems. In this section, I discuss three problems, initiated by operators, that I worked on while I was the OEG Field Representative at the Whidbey Island (Washington) Naval Air Station. The aircraft operating out of Whidbey are the medium attack A-6 bomber and the EA-6B electronic warfare aircraft.

5.1. EA-6B pod availability. The EA-6B carries its jamming equipment in pods that are mounted on the wing stations. There are two transmitters per pod and each aircraft can carry up to five pods. Ideally, one would like all pods to work all the time. I shall refer to the overall fraction of the time that a pod works as the availability of the pod. If the availability is low, then it is important to understand the causes of poor availability and to devise ways to improve it.

Within a few weeks of my arrival at Whidbey, in February, 1979, a squadron returned home from a cruise with data on pod availability. They had set some simple criteria for a pod's being classified as working and had counted the fraction of pods that worked. They found low availability. These numbers were reported to the Admiral, who wanted further investigation. He tasked an officer, who came to me with the log book that the squadron had used and asked me to sift through and analyze the data.

When the data were first reported, as "a certain type of pod worked 70% of the time," one infers a picture of pod availability. The picture in this case is flipping a coin: a simple Bernoulli trial. As I analyzed the data, it became clear that a more complex picture was needed. I developed a series of Markov models for pod availability, the simplest of which is the following.

Assume that there are only two states for a pod, mission capable (MC) or not mission capable (NMC), and let $P_1(n)$, $P_0(n)$ be the probabilities that a pod is MC or NMC on the nth flight. Let p_{11}, p_{10}, p_{00}, p_{01} be the probability of transition MC $\rightarrow$ MC, MC $\rightarrow$ NMC, NMC $\rightarrow$ NMC, NMC $\rightarrow$ MC respectively. Then

$$P_1(n+1) = P_1(n)p_{11} + P_0(n)p_{01}. \tag{1}$$

Since $P_0(n) = 1 - P_1(n)$, equation (1) becomes

$$P_1(n+1) = p_{01} + P_1(n)(p_{11} - p_{01}). \tag{2}$$

The stationary solution of (2) is

$$P_{1s} = \frac{p_{01}}{p_{10} + p_{01}}. \tag{3}$$

The p_{i1}, which can be estimated from the data, have the following interpretations: p_{01} is a measure of the effectiveness of maintenance, since it characterizes the rate at which pods that are NMC return to the MC state; p_{11} is a measure of reliability, since it characterizes the probability that that a pod which is MC remains MC. The long term availability can be low if either p_{01} or p_{11} is low; however, ways to increase availability differ very much, depending on whether the problem is in maintenance or reliability. If p_{01} is small, so that maintenance is the cause of poor availability, it may be possible to improve availability through increased attention to the problems of maintenance. Thus one is faced with a *command* problem. If p_{11} is small, however, the problem of poor availability is related to the design of the equipment and improving availability may no longer be connected with operational aspects. I found that p_{01} was about .2 or less, whereas p_{11} was about .8 or more, and suggested that availability could probably be improved by command attention to maintenance.

I checked other predictions of the simple two-state Markov model against the data and found that this simple model stood up well to scrutiny; so I presented my results to the command. I argued, and convinced officers, that the picture of the operations must be changed. Pod status is no longer anologous to flipping a coin; more complicated verbal models are needed.

After the completion of this initial work, we began an in-depth study of the causes of low pod availability. We started using a four-state Markov chain model and collected data from 7 of the 10 EA-6B squadrons that go to the fleet. Along with operational data, collected on aircrew debrief sheets, we had maintenance data sent back to Whidbey. I continued my analysis of these data, using any mathematical or statistical technique that I thought helpful.

By May, 1980, I was able to prepare a fairly substantial report on causes of poor pod availability. This report included information on the overall availability of pods, relationship between pod availability and aircraft, conclusions about reliability, and causes of poor maintenance and ways to improve maintenance. Pieces of this report appeared in various newsletters; I presented my results to the EA-6B design review conference in August, 1980, and the report was sent to VADM McDonald, the Deputy Chief of Naval Operations for Air Warfare. This work certainly led to a change in the way the problem was viewed; whether the overall availability will increase due to recommendations being followed remains to be seen.

5.2. Wings level landings on angle decks. I became involved in this problem because I shared an office with an experienced A-6 pilot and overheard a conversation. My office

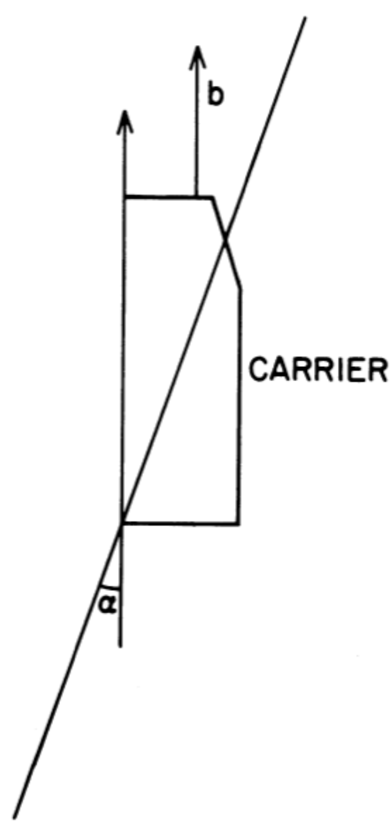

FIG. 2. *Geometry for the angle deck landing problem.*

mate was arguing with two new pilots about the possibility of a "wings level" landing on a carrier deck. The young pilots asserted that in flight training they were given a proof that it was possible to line up the aircraft with the landing deck of the carrier and then drive the ship with the proper velocity so that a pilot could land without making another correction. The older pilot did not believe these claims, even when he saw the proof. So he asked me to take a look at the problem.

The relevant geometry is sketched in Fig. 2. A vector parallel to the angle deck is $I = (1, -1/\tan\alpha)$; a characteristic value is $\alpha = 10°$. In typical operations, the ship is driven so that the resultant of ship's motion and natural wind is "straight down the angle," i.e., parallel to I, with a magnitude of about 30 knots (kt.). Suppose that the ship's velocity vector is $b = (b_x, b_y)$ and the wind is $w = (w_x, -w_y)$ so that the resultant wind is $r = (w_x - b_x, -w_y - b_y)$. We now want to drive the ship so that r is parallel to I and so that the magnitude of r is u. This gives the following equations for b:

$$(4)\qquad \begin{aligned} b_x &= w_x \mp \frac{u}{\left[1 + \dfrac{1}{\tan^2\alpha}\right]^{1/2}}, \\ b_y &= -w_y \pm \frac{1}{\tan\alpha}\frac{u}{\left[1 + \dfrac{1}{\tan^2\alpha}\right]^{1/2}}. \end{aligned}$$

Table 1 shows the ship's speed needed to keep a 30 kt wind down a 10° angle. From Table 1 we conclude that it is possible to drive the ship properly, but that as the wind increases, the speed of the ship becomes unreasonably large.

Thus it appears that wings level landings are at least theoretically possible in a nonfluctuating world. Thus, let us consider now the effects of fluctuations in the velocity of the ship and aircraft. In such a case, we can only ask for the probability of a wings level landing. To do this, let $(S_x(t), S_y(t))$ and $(A_x(t), A_y(t))$ denote the positions of the ship and aircraft respectively (assuming that they already are in the same plane). We assume that these variables satisfy the equations

$$(5)\qquad \begin{aligned} \dot{S}_x &= b_x + \xi_1(t), & \dot{A}_x &= w_x - v_x + \xi_3(t), \\ \dot{S}_y &= b_y + \xi_2(t), & \dot{A}_y &= w_y - v_y + \xi_4(t), \\ S_x(0) &= S_y(0) = 0, & A_x(0) &= a_x, \quad A_y(0) = a_y. \end{aligned}$$

In (5), the $\xi_i(t)$ are independent Gaussian white noise processes, (w_x, w_y) is the wind

TABLE 1
Ship's speed needed to keep a 30-kt wind down the angle

Wind speed (kt.)	Direction	Ship's speed
10	30	28
	45	26
	60	23
20	30	30
	45	25
	60	19
30	30	35
	45	28
	60	21

vector, (v_x, v_y) is the velocity vector of the aircraft and (a_x, a_y) is the initial position of the aircraft. We assume that a_x and a_y are picked so that the aircraft is lined up with the angle deck; thus $a_x = a_y \tan\alpha$. Furthermore, we assume that v_x and v_y are picked so that deterministically the aircraft can make a wings level landing. To find them, we drop the noise terms in (5), and set $S_x(t) = A_x(t)$ and $S_y(t) = A_y(t)$ to obtain the following equation for v_x:

$$(b_y + \sqrt{v^2 - v_x^2} - w_y)\tan\alpha = b_x + v_x - w_x. \tag{6}$$

In (6), v is the speed of the aircraft; once v_x is known, $v_y = (v^2 - v_x^2)^{1/2}$.

Let us proceed with a "best case" calculation of the probability of a wings level landing. Thus, we assume that b_x, b_y, ξ_1, ξ_2, w_x, and w_y are identically zero. In this case, the only noise is due to fluctuations in aircraft position. From (6) it is easy to see that $v_x = v/(1 + \tan^{-2}\alpha)^{1/2}$ and that $v_y = v_x/\tan\alpha$. Since the integral of white noise is normally distributed, we conclude from (5) that $A_x(t)$ is normally distributed with mean and variance

$$\mu_x(t) = a_x - v_x t, \tag{7}$$

$$\sigma_x^2(t) = \sigma_1^2 t, \tag{8}$$

where $\sigma_1^2 t$ is the variance of the Wiener process obtained by integrating $\xi_1(t)$. Similarly, $A_y(t)$ is normally distributed with mean and variance

$$\mu_y(t) = a_y - v_y t, \tag{9}$$

$$\sigma_y^2(t) = \sigma_2^2 t. \tag{10}$$

In this example, the ship stays at the origin. Thus, the probability of a wings level landing is the joint probability that $A_x(t)$ and $A_y(t)$ are simultaneously approximately zero. By approximately, let us understand that they are within $2l$ of zero. The probability of a wings level landing P_L can then be defined by

$$P_L = \max_t \left\{\int_{-l}^{l} e^{-(x-\mu_x(t))^2/2\sigma_x^2(t)} \frac{dx}{\sqrt{2\pi\sigma_x^2(t)}}\right\}\left\{\int_{-l}^{l} e^{-(y-\mu_y(t))^2/2\sigma_y^2(t)} \frac{dy}{\sqrt{2\pi\sigma_y^2(t)}}\right\}. \tag{11}$$

In Table 2, values of P_L are given for these parameters: $a_y = 1$ nautical mile (n.mi.), $v = 120$ kt, $\alpha = 10°$ and various values of σ_1 and σ_2. We assume that they are equal, so that $\sigma_1 = \sigma_2 = \sigma$, and define the intensity of fluctuations by $(\sigma_1^2 + \sigma_2^2)t_d/(a_x^2 + a_y^2)^{1/2}$, where t_d is the deterministic time to land the aircraft.

When considering the results shown in Table 2, one should also know that aviators

TABLE 2
Probability of a wings level landing

Intensity of fluctuations	Probability of a wings level landing
0	1
1.67×10^{-6}	.97
4.17×10^{-5}	.11
1.67×10^{-4}	.024
4.17×10^{-3}	1.04×10^{-3}
.0167	2.7×10^{-4}
.0667	6.7×10^{-5}
.15	2.9×10^{-5}

are given badges for 100 landings on a carrier. Thus an event that has a probability of occurrence of 10^{-4} at each landing is rarely going to be observed in any aviator's career. The conclusion is that wings level landings are possible, but highly improbable in the world modelled by (5).

5.3. Three-bearing method for passive localization. One of the missions of the EA-6B is that of passive localization of surface radar emitters. This localization is done by some sort of triangulation. The observed bearing between aircraft and emitter, θ_0, is given by

$$\theta_0 = \theta_r + b(\theta_r) + \varepsilon \tag{12}$$

where θ_r is the real bearing between the aircraft and emitter, $b(\theta_r)$ is a deterministic, but unknown bias function, and ε is a random noise term. For the EA-6B, $|b(\theta_r)|$ may be as large as 30°, while the root mean square deviation of ε is about 1°–3°.

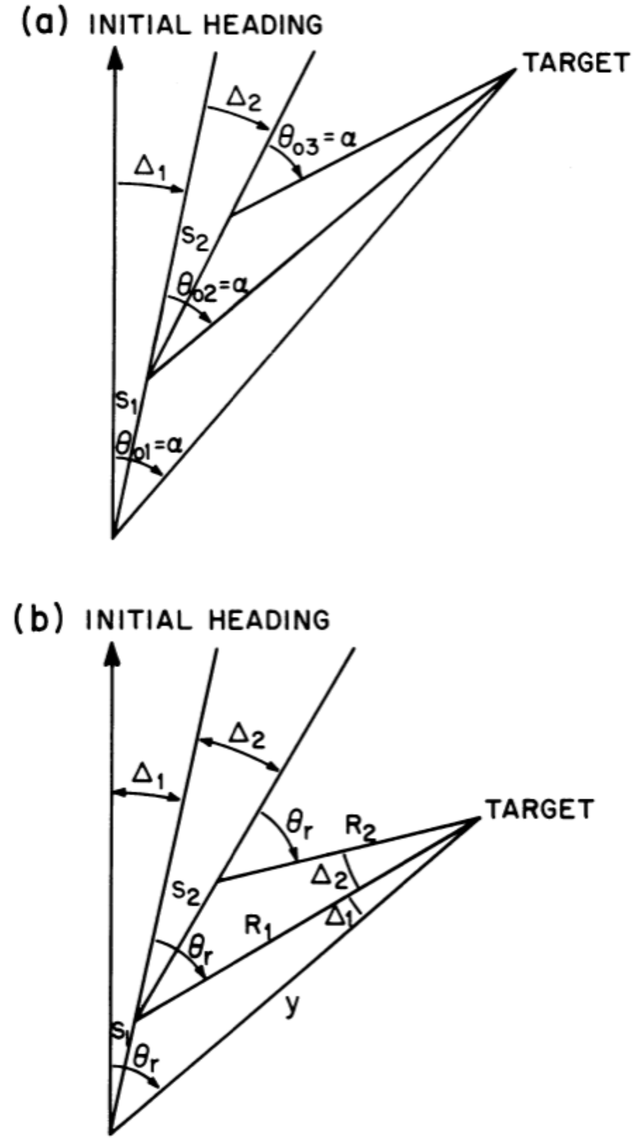

FIG. 3. *Characterization of the three-bearing method.*

Since about 1945, triangulation problems with random noise received much attention (e.g., [12], [13]), while the questions of the unknown deterministic bias were ignored. In the EA-6B, at least, the deterministic bias overrides the random noise in almost all cases, and yet until recently there was no way to treat it. In collaboration with two officers, an EA-6B operator and an A-6 pilot, I developed the following method for localization in systems with an unknown deterministic bias [14]. Its operational description is the following:

1. An initial value of the observed bearing, $\theta_{01} = \alpha$, is recorded.
2. The aircraft turns through an angle Δ_1. If the observed bearing is positive (negative) then the turn is positive (negative). The aircraft flies until the observed bearing θ_{02} is equal to α. The distance travelled, s_1, is recorded.
3. The aircraft turns again, through an angle Δ_2, and flies until the observed bearing θ_{03} is equal to α. The distance travelled, s_2, is recorded.

The geometry associated with this operation is shown in Fig. 3. Referring to Fig. 3 and using the law of sines twice shows that

$$\frac{s_1}{\sin \Delta_1} = \frac{y}{\sin \theta_r} = \frac{R_1}{\sin (\theta_r - \Delta_1)}, \qquad \frac{s_2}{\sin \Delta_2} = \frac{R_1}{\sin \theta_r} = \frac{R_2}{\sin (\theta_r - \Delta_2)}. \tag{13}$$

For the triangle with sides s_1, y, and R_1,

$$\cos \Delta_1 = \frac{y^2 + R_1^2 - s_1^2}{2yR_1}. \tag{14}$$

From (8)

$$y = \frac{s_1 \sin \theta_r}{\sin \Delta_1} \quad \text{and} \quad \sin \theta_r = \frac{R_1 \sin \Delta_2}{s_2}, \tag{15}$$

so that

$$y = \frac{s_1}{s_2} \cdot R_1 \cdot \frac{\sin \Delta_2}{\sin \Delta_1} = \gamma \eta R_1; \tag{16}$$

here $\gamma = s_1 s_2$, $\eta = \sin \Delta_2 / \sin \Delta_1$.

Using (16) in (14) gives

$$\cos \Delta_1 = \frac{\gamma^2 \eta^2 R_1^2 + R_1^2 - s_1^2}{2\gamma\eta R_1^2}. \tag{17}$$

Equation (17) is cast into a dimensionless form by setting $r_1 = R_1/s_2$. Then, after some simplification, (17) becomes

$$r_1 = \frac{\gamma}{[\gamma^2 \eta^2 + 1 - 2\gamma\eta \cos \Delta_1]^{1/2}}. \tag{18}$$

From (15), $\sin \theta_r = r_1 \sin \Delta_2$, so that $\theta_r = \arcsin (r_1 \sin \Delta_2)$. Since the arc sin is double valued, more work is needed to evaluate θ_r uniquely. In [14], it is shown that by switching to Cartesian coordinates, one finds that

$$\tan \theta_r = \frac{[-s_2 \tan \Delta_2 \tan (\Delta_1 + \Delta_2) - (s_1 \sin \Delta_2 - (s_2 + s_1 \cos \Delta_2) \tan (\Delta_1 + \Delta_2)) \tan \Delta_2]}{[-s_2 \tan (\Delta_1 + \Delta_2) + \tan \Delta_2 (s_2 + s_1 \cos \Delta_2 + s_1 \sin \Delta_2 \tan (\Delta_1 + \Delta_2))]}, \tag{19}$$

so that θ_r is known uniquely. Once θ_r is known, from (13) we find that

$$R_2 = \frac{R_1 \sin(\theta_r - \Delta_2)}{\sin \theta_r}. \tag{20}$$

This method was flight tested in the summer of 1980 and was found to be operationally feasible and accurate to within 10% or less in range error. Thus, this method allows triangulation in systems with an unknown bias. In [14], it is shown how random errors can be incorporated into the method.

6. Conclusion. There are many problems and problem areas not mentioned in the above examples. A different analyst, by nature of taste and personality, would probably have worked on different problems, but I believe that the general principles are applicable to any field in which mathematics is to be applied. In addition to the principles listed in §2, I believe that it is important to recognize the prevalence of stochastic effects and of unknown deterministic biases in the natural world. Too much analysis is done based on assumptions of a perfectly predictable world, in which plans can be made at the outset and need not be changed.

Honesty requires that I point out some of the frustrations of this kind of work. The first is a lack of continuity. With good reason, an analyst should be at a field assignment no longer than about 2 or 3 years. Otherwise he becomes stale and too emotionally involved with the job to retain a high level of objectivity. But there is no reason to expect that one's replacement will follow up on projects that have been started. In general, that will not happen and it can be disappointing. The second frustration is that, as with all jobs, there is drudgery in this one also. Not every day contains excitement. A third frustration is that often there is little feedback about the analysis one has done. The analyst finds himself forcing people to read his work and respond to it. Sometimes this is an unpleasant task. These frustrations appear in almost any job, and surely some would appear in any academic job.

The principles that make OEG effective undoubtedly will work in any type of nonacademic setting, and point the way for mathematicians to be effective in the use of their tools.

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