Appendices

Appendix I - Derivation of $\mathrm{E}\{N(\Omega)\}_{\mathrm{RV}}$: If κ is treated as a random variable, such that the probability that $K = \kappa$ varies according to a truncated Poisson distribution, considering branches ≥ 2 with a mean value of λ, following the format of eqn (2), the expected number of nodes can be calculated as

$$\mathrm{E}\{N\}_{\mathrm{RV}} = \begin{cases} \sum_{\omega=1}^{\Omega} \left(p \times \sum_{\kappa=2}^{\infty} \kappa \frac{\mathrm{e}^{-\lambda}\lambda^{\kappa}}{\kappa!(1-Q)} + (1-p) \times 1 \right)^{\omega-1} & \text{for } 0 < p \leq 1 \\ \Omega & \text{for } p = 0, \end{cases} \tag{1}$$

The part of this equation corresponding to $0 < p \leq 1$ can be simplified, as

$$\sum_{\kappa=2}^{\infty} \frac{\kappa\lambda^{\kappa}}{\kappa!} = \lambda \left(\sum_{\kappa=0}^{\infty} \frac{\lambda^{\kappa}}{\kappa!} - 1 \right) = \lambda(\mathrm{e}^{\lambda} - 1). \tag{2}$$

Therefore, we obtain

$$\mathrm{E}\{N\}_{\mathrm{RV}} = \sum_{\omega=1}^{\Omega} \left(\frac{p\mathrm{e}^{-\lambda}\lambda(\mathrm{e}^{\lambda} - 1)}{1-Q} + 1 - p \right)^{\omega-1} = \sum_{\omega=1}^{\Omega} \left(\frac{p\lambda(1 - \mathrm{e}^{-\lambda})}{1-Q} + 1 - p \right)^{\omega-1}. \tag{3}$$

If you call

$$\Psi = \frac{p\lambda(1 - \mathrm{e}^{-\lambda})}{1-Q} + 1 - p, \tag{4}$$

then

$$\mathrm{E}\{N\}_{\mathrm{RV}} = \frac{\Psi^{\Omega} - 1}{\Psi - 1}. \tag{5}$$

Appendix II - Derivation of $\mathrm{CV}_a(\omega)$: First, we set the dynamics of a population inhabiting a tributary node (a node at the order of observation $\omega = \Omega$) to have a coefficient of variation $\mathrm{CV}_a(\omega = \Omega) = \mathrm{CV}_\Omega$.

For $\omega < \Omega$ we set

$$\mathrm{CV}_a(\omega) = \mathrm{CV}_a(\omega + 1) \times \underbrace{\left[\frac{1 + r(N_{\mathrm{inflow}}(\omega) - 1)}{N_{\mathrm{inflow}}(\omega)}\right]^{1/2}}_{G(\omega)}, \tag{1}$$

where $N_{\mathrm{inflow}}(\omega)$ denotes the number of nodes flowing directly into the node at the order of observation ω. If the branch number is a constant, the number of inflowing nodes is simply κ with probability p, and 1 with probability $(1-p)$. In the probabilistic river metapopulation, where the number of branches treated as a random variable, the number of inflowing nodes is calculated as

$$\mathrm{E}\{N_{\mathrm{inflow}}(\omega)\} = p \times \sum_{\kappa=2}^{\infty} \kappa \frac{\mathrm{e}^{-\lambda}\lambda^{\kappa}}{\kappa!(1-Q)} + (1-p) \times 1, \tag{2}$$

where Q is the regularized incomplete gamma function. Note also that this quantity is equivalent to Ψ, which we define in Appendix I as

$$\Psi = \frac{p\lambda(1 - \mathrm{e}^{-\lambda})}{1 - Q} + 1 - p, \tag{3}$$

where λ is the expected branching number given κ is treated as a random variable K, such that $\mathrm{E}\{K\} = \lambda$.

For efficiency, we refer to the term to the right of the $\times$ symbol in eqn (1) as the function $G(\omega)$. The downstream node at network order ω is a blend of populations inhabiting upstream nodes, the number of which depend on the branch number κ. We can determine the CV for downstream nodes recursively, such that

$$\mathrm{CV}_a(\Omega - 1) = \mathrm{CV}_\Omega \times G(\Omega - 1),$$

$$\mathrm{CV}_a(\Omega - 2) = \mathrm{CV}(\Omega - 1) \times G(\Omega - 2),$$

$$\vdots$$

$$\mathrm{CV}_a(1) = \mathrm{CV}(2) \times G(1),$$

allowing us to write more generally

$$\mathrm{CV}_a(\omega) = \mathrm{CV}_\Omega \left(\frac{1 + r(\Psi - 1)}{\Psi} \right)^{\frac{\Omega - \omega}{2}} . \tag{4}$$